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No interactions for a collection of spin-two fields intermediated by a massive Rarita–Schwinger field

C. Bizdadea[a], E.M. Cioroianu[b], D. Cornea[c], S.O. Saliu[d], S.C. Săraru[e]

Faculty of Physics, University of Craiova, 13 A.I. Cuza Str., Craiova 200585, Romania

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Abstract. The cross-couplings among several massless spin-two fields (described in the free limit by a sum of Pauli–Fierz actions) in the presence of a massive Rarita–Schwinger field are investigated in the framework of the deformation theory based on local BRST cohomology. Under the hypotheses of locality, smoothness of the interactions in the coupling constant, Poincaré invariance, Lorentz covariance, and the preservation of the number of derivatives on each field, we prove that there are no consistent cross-interactions among different gravitons with a positively defined metric in internal space in the presence of a massive Rarita–Schwinger field. The basic features of the couplings between a single Pauli–Fierz field and a massive Rarita–Schwinger field are also emphasized.

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1 Introduction

Over the last twenty years there has been a sustained effort to construct theories involving a multiplet of spin-two fields [1–4]. At the same time, various couplings of a single massless spin-two field to other fields (including itself) have been studied in [5–15]. In this context, the impossibility of cross-interactions among several Einstein gravitons under certain assumptions was proved recently in [16] by means of a cohomological approach based on the Lagrangian BRST symmetry [17–21]. Moreover, in [16], the impossibility of cross-interactions among different Einstein gravitons in the presence of a scalar field has also been shown.

The main aim of this paper is to investigate the cross-couplings among several massless spin-two fields (described in the free limit by a sum of Pauli–Fierz actions) in the presence of a massive Rarita–Schwinger field. More precisely, under the hypotheses of locality, smoothness of the interactions in the coupling constant, Poincaré invariance, (background) Lorentz invariance, and the preservation of the number of derivatives on each field, we prove that there are no consistent cross-interactions among different gravitons with a positively defined metric in internal space in the presence of a massive Rarita–Schwinger field. This result is obtained by using the deformation technique [22] combined with the local BRST cohomology [23]. It is a well-known fact that the spin-two field in metric formulation (Einstein–Hilbert theory) cannot be coupled to a spin-3/2 field. However, as will be shown below, if we decompose the metric as $g_{\mu\nu} = \sigma_{\mu\nu} + \lambda h_{\mu\nu}$, where $\sigma_{\mu\nu}$ is the flat metric and λ is the coupling constant, we can indeed couple the massive spin-3/2 field to $h_{\mu\nu}$ in the space of formal series with the maximum derivative order equal to one in $h_{\mu\nu}$. Thus, our approach envisages two different aspects. One is related to the couplings between the spin-two fields and one massive Rarita–Schwinger field, while the other focuses on proving the impossibility of cross-interactions among different gravitons via a single massive Rarita–Schwinger field. In order to make the analysis as clear as possible, we initially consider the case of the couplings between a single Pauli–Fierz field [24] and a massive Rarita–Schwinger field [25]. In this setting, we compute the interaction terms to order two in the coupling constant. Next, we prove the isomorphism between the local BRST cohomologies corresponding to the Pauli–Fierz theory and to the linearized version of the vierbein formulation of the spin-two field, respectively. Since the deformation procedure is controlled by the local BRST cohomology of the free theory (in ghost numbers zero and one), the previous isomorphism allows us to translate the results emerging from the Pauli–Fierz formulation into the vierbein version and conversely. In this manner, we obtain that the first two orders of the interacting Lagrangian resulting from our setting originate in the development of the full interacting Lagrangian

$$\mathcal{L}^{(\mathrm{int})} = \frac{e}{2}\left(-\mathrm{i}\bar{\psi}_\mu e_a{}^\mu e_b{}^\nu e_c{}^\rho \gamma^{abc} D_\nu \psi_\rho + m\bar{\psi}_\mu e_a{}^\mu \gamma^{ab} e_b{}^\nu \psi_\nu\right) + \lambda\left[eV(X,Y,Z) + d_1(X,Y,Z)\, e_a{}^\nu \bar{\psi}_\nu \gamma^a D_\mu (e\psi^\mu) + ed_2(X,Y,Z)\left(\bar{\psi}^\mu \gamma^b + e_a{}^\mu e^b{}_\rho \bar{\psi}^\rho \gamma^a\right) D_\mu\left(e_b{}^\nu \psi_\nu\right)\right].$$

[a] e-mail: bizdadea@central.ucv.ro
[b] e-mail: manache@central.ucv.ro
[c] e-mail: danubiusmd@yahoo.com
[d] e-mail: osaliu@central.ucv.ro
[e] e-mail: scsararu@central.ucv.ro

Here, ${e_a}^{\mu}$ represent the vierbein fields, e is the inverse of their determinant, $e = \left(\det\left({e_a}^{\mu}\right)\right)^{-1}$, D_μ signifies the full covariant derivative, and γ^a stand for the flat Dirac matrices. The fields ψ_ν denote the (curved) Rarita–Schwinger spinors ($\psi_\nu = e^a{}_\nu \psi_a$). The quantities denoted by V, d_1, and d_2 are arbitrary polynomials of $X \equiv \bar{\psi}_a \psi^a$, $Y \equiv \bar{\psi}_a \gamma^{ab} \psi_b$, and $Z = \mathrm{i}\bar{\psi}_a \gamma_5 \psi^a$. Here and in the sequel λ is the coupling constant (deformation parameter). We observe that the first two terms in $\mathcal{L}^{(\mathrm{int})}$ describe the standard minimal couplings between the spin-two and massive Rarita–Schwinger fields. The last terms from $\mathcal{L}^{(\mathrm{int})}$, namely those proportional to V, d_1, or d_2, produce non-minimal couplings. To our knowledge, these non-minimal interaction terms are not discussed in the literature. However, they are consistent with the gauge symmetries of the Lagrangian $\mathcal{L}_2 + \mathcal{L}^{(\mathrm{int})}$, where $\mathcal{L}_2$ is the full spin-two Lagrangian in the vierbein formulation. With this result at hand, we start from a finite sum of Pauli–Fierz actions with a positively defined metric in internal space and a massive Rarita–Schwinger field, and prove that there are no consistent cross-interactions between different gravitons in the presence of such a fermionic matter field.

This paper is organized in seven sections. In Sect. 2 we construct the BRST symmetry of a free model with a single Pauli–Fierz field and one massive Rarita–Schwinger field. Section 3 briefly addresses the deformation procedure based on BRST symmetry. In Sect. 4 we compute the first two orders of the interactions between one graviton and one massive Rarita–Schwinger spinor. Section 5 presents the Lagrangian formulation of the interacting theory. Section 6 is devoted to the proof of the fact that there are no consistent cross-interactions among different gravitons in the presence of a massive Rarita–Schwinger field. Section 7 exposes the main conclusions of the paper. The present paper also contains two appendices, in which various notations and conditions are listed and also some statements from the body of the paper are proved.

2 Free model: Lagrangian formulation and BRST symmetry

Our starting point is represented by a free model, whose Lagrangian action is written like the sum between the action of the linearized version of Einstein–Hilbert gravity (the Pauli–Fierz action [24]) and that of a massive Rarita–Schwinger field [25]

$$
\begin{aligned}
S_0^{\mathrm{L}}\left[h_{\mu\nu}, \psi_\mu\right] = \int \mathrm{d}^4x \Big(& -\frac{1}{2}\left(\partial_\mu h_{\nu\rho}\right)\left(\partial^\mu h^{\nu\rho}\right) \\
& + \left(\partial_\mu h^{\mu\rho}\right)\left(\partial^\nu h_{\nu\rho}\right) - \left(\partial_\mu h\right)\left(\partial_\nu h^{\nu\mu}\right) \\
& + \frac{1}{2}\left(\partial_\mu h\right)\left(\partial^\mu h\right) - \frac{\mathrm{i}}{2}\bar{\psi}_\mu \gamma^{\mu\nu\rho}\partial_\nu \psi_\rho \\
& + \frac{m}{2}\bar{\psi}_\mu \gamma^{\mu\nu}\psi_\nu \Big) \\
\equiv \int d^4x \Big(& \mathcal{L}^{(\mathrm{PF})} + \mathcal{L}_0^{(\mathrm{RS})} \Big) \\
= S_0^{\mathrm{PF}} & \left[h_{\mu\nu}\right] + S_0^{\mathrm{RS}}\left[\psi_\mu\right] . \qquad (1)
\end{aligned}
$$

Everywhere in this paper we use the flat Minkowski metric of 'mostly minus' signature, $\sigma_{\mu\nu} = (+---)$. In the above, h denotes the trace of the Pauli–Fierz field, $h = \sigma_{\mu\nu}h^{\mu\nu}$, and the fermionic fields ψ_μ are considered to be real (Majorana) spinors. We work with a representation of the Clifford algebra

$$
\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\sigma_{\mu\nu}\mathbf{1}\,, \qquad (2)
$$

in which all the γ matrices are purely imaginary, so that we have

$$
\gamma_\mu^{\mathsf{T}} = -\gamma_0 \gamma_\mu \gamma_0\,, \quad \mu = \overline{0,3}\,, \qquad (3)
$$

where here and in the sequel the notation N^{T} signifies the transpose of the matrix N. In addition, γ_0 is Hermitian and antisymmetric, while $(\gamma_i)_{i=\overline{1,3}}$ are anti-Hermitian and symmetric. The Dirac conjugation is defined as usual through

$$
\bar{\psi}_\mu = \left(\psi_\mu\right)^{\dagger}\gamma_0\,, \qquad (4)
$$

and the Majorana conjugation via

$$
\psi^c = \left(\mathcal{C}\psi\right)^{\mathsf{T}}\,, \qquad (5)
$$

with the corresponding charge conjugation given by

$$
\mathcal{C} = -\gamma_0\,. \qquad (6)
$$

(The operation † signifies the Hermitian conjugation.) Action (1) possesses an irreducible and Abelian generating set of gauge transformations

$$
\delta_\epsilon h_{\mu\nu} = \partial_{(\mu}\epsilon_{\nu)}\,, \quad \delta_\epsilon \psi_\mu = 0\,, \qquad (7)
$$

with ϵ_μ being bosonic gauge parameters. The parentheses signify symmetrization; they are never divided by the number of terms: e.g., $\partial_{(\mu}\epsilon_{\nu)} = \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu$, and the minimum number of terms is always used. The same is valid with respect to the notation $[\mu \cdots \nu]$, which means antisymmetrization with respect to the indices between brackets.

In order to construct the BRST symmetry for (1), we introduce the fermionic ghosts η_μ corresponding to the gauge parameters ϵ_μ and associate antifields with the original fields and ghosts, respectively denoted by $\left\{h^{*\mu\nu}, \psi^*_\mu\right\}$ and $\left\{\eta^{*\mu}\right\}$. (The statistics of the antifields is opposite to that of the correlated fields/ghosts.) The antifields of the Rarita–Schwinger fields are bosonic, purely imaginary spinors. Since the gauge generators of the free theory under study are field independent and irreducible, it follows that the BRST differential simply decomposes into

$$
s = \delta + \gamma\,, \qquad (8)
$$

where δ represents the Koszul–Tate differential, graded by the antighost number agh (agh $(\delta) = -1$), and γ stands for the exterior derivative along the gauge orbits, whose degree is named pure ghost number pgh (pgh $(\gamma) = 1$). These two degrees do not interfere (pgh $(\delta) = 0$, agh $(\gamma) = 0$). The overall degree from the BRST complex is known as the ghost number gh and is defined like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}(\delta) = \mathrm{gh}(\gamma) = \mathrm{gh}(s) = 1$. If we make the notations

$$\Phi^{\alpha_0} = (h_{\mu\nu}, \psi_\mu) , \quad \Phi^*_{\alpha_0} = \left(h^{*\mu\nu}, \psi^*_\mu\right) , \tag{9}$$

then, according to the standard rules of the BRST formalism, the degrees of the BRST generators are valued like

$$\mathrm{agh}\left(\Phi^{\alpha_0}\right) = \mathrm{agh}\left(\eta_\mu\right) = 0 , \quad \mathrm{agh}\left(\Phi^*_{\alpha_0}\right) = 1 ,$$
$$\mathrm{agh}\left(\eta^{*\mu}\right) = 2 , \tag{10}$$
$$\mathrm{pgh}\left(\Phi^{\alpha_0}\right) = 0 , \quad \mathrm{pgh}\left(\eta_\mu\right) = 1 ,$$
$$\mathrm{pgh}\left(\Phi^*_{\alpha_0}\right) = \mathrm{pgh}\left(\eta^{*\mu}\right) = 0 . \tag{11}$$

The actions of the differentials δ and γ on the generators from the BRST complex are given by

$$\delta h^{*\mu\nu} = 2H^{\mu\nu}, \;\; \delta\psi^{*\mu} = m\bar{\psi}_\lambda\gamma^{\lambda\mu} - \mathrm{i}\partial_\rho\bar{\psi}_\lambda\gamma^{\rho\lambda\mu} , \tag{12}$$
$$\delta\eta^{*\mu} = -2\partial_\nu h^{*\mu\nu} , \tag{13}$$
$$\delta\Phi^{\alpha_0} = 0 = \delta\eta_\mu , \tag{14}$$
$$\gamma\Phi^*_{\alpha_0} = 0 = \gamma\eta^{*\mu} , \tag{15}$$
$$\gamma h_{\mu\nu} = \partial_{(\mu}\eta_{\nu)}, \quad \gamma\psi_\mu = 0, \quad \gamma\eta_\mu = 0 , \tag{16}$$

where $H^{\mu\nu}$ is the linearized Einstein tensor

$$H^{\mu\nu} = K^{\mu\nu} - \frac{1}{2}\sigma^{\mu\nu}K , \tag{17}$$

with $K^{\mu\nu}$ and K the linearized Ricci tensor and, respectively, the linearized scalar curvature, both obtained from the linearized Riemann tensor

$$K_{\mu\nu\alpha\beta} = -\frac{1}{2}\left(\partial_\mu\partial_\alpha h_{\nu\beta} + \partial_\nu\partial_\beta h_{\mu\alpha} - \partial_\nu\partial_\alpha h_{\mu\beta} - \partial_\mu\partial_\beta h_{\nu\alpha}\right) , \tag{18}$$

via its trace and double trace, respectively,

$$K_{\mu\alpha} = \sigma^{\nu\beta}K_{\mu\nu\alpha\beta} , \quad K = \sigma^{\mu\alpha}\sigma^{\nu\beta}K_{\mu\nu\alpha\beta} . \tag{19}$$

The BRST differential is known to have a canonical action in a structure named antibracket and denoted by the symbol $(,)$ $(s\cdot = (\cdot, \bar{S}))$, which is obtained by decreeing the fields/ghosts conjugated to the corresponding antifields. The generator of the BRST symmetry is a bosonic functional of ghost number zero, which is the solution to the classical master equation $(\bar{S}, \bar{S}) = 0$. The full solution to the classical master equation for the free model under study reads as

$$\bar{S} = S_0^{\mathrm{L}}\left[h_{\mu\nu}, \psi_\mu\right] + \int \mathrm{d}^4x\, h^{*\mu\nu}\partial_{(\mu}\eta_{\nu)} . \tag{20}$$

3 Deformation of the solution to the master equation: a brief review

We begin with a "free" gauge theory, described by a Lagrangian action $S_0^{\mathrm{L}}[\Phi^{\alpha_0}]$, invariant under some gauge transformations $\delta_\epsilon\Phi^{\alpha_0} = Z^{\alpha_0}_{\alpha_1}\epsilon^{\alpha_1}$, i.e. $\frac{\delta S_0^{\mathrm{L}}}{\delta\Phi^{\alpha_0}}Z^{\alpha_0}_{\alpha_1} = 0$, and consider the problem of constructing consistent interactions among the fields Φ^{α_0} such that the couplings preserve both the field spectrum and the original number of gauge symmetries. This matter is addressed by means of reformulating the problem of constructing consistent interactions as a deformation problem of the solution to the master equation corresponding to the "free" theory [22]. Such a reformulation is possible due to the fact that the solution to the master equation contains all the information on the gauge structure of the theory. If an interacting gauge theory can be consistently constructed, then the solution $\bar{S}$ to the master equation $(\bar{S}, \bar{S}) = 0$ associated with the "free" theory can be deformed into a solution S

$$\bar{S} \to S = \bar{S} + \lambda S_1 + \lambda^2 S_2 + \ldots = \bar{S} + \lambda\int \mathrm{d}^Dx\, a + \lambda^2\int \mathrm{d}^Dx\, b + \ldots , \tag{21}$$

of the master equation for the deformed theory

$$(S, S) = 0 , \tag{22}$$

such that both the ghost and antifield spectra of the initial theory are preserved. Equation (22) splits, according to the various orders in the coupling constant (deformation parameter) λ, into a tower of equations:

$$(\bar{S}, \bar{S}) = 0 , \tag{23}$$
$$2(S_1, \bar{S}) = 0 , \tag{24}$$
$$2(S_2, \bar{S}) + (S_1, S_1) = 0 , \tag{25}$$
$$(S_3, \bar{S}) + (S_1, S_2) = 0 , \tag{26}$$
$$\vdots$$

Equation (23) is fulfilled by the hypothesis. The next equation requires that the first-order deformation of the solution to the master equation, S_1, is a cocycle of the "free" BRST differential $s\cdot = (\cdot, \bar{S})$. However, only cohomologically non-trivial solutions to (24) should be taken into account, as the BRST-exact solutions can be eliminated by some (in general non-linear) field redefinitions. This means that S_1 pertains to the ghost number zero cohomological space of s, $H^0(s)$, which is generically non-empty because it is isomorphic to the space of physical observables of the "free" theory. It has been shown (by the triviality of the antibracket map in the cohomology of the BRST differential) that there are no obstructions in finding solutions to the remaining equations, namely (25), (26), etc. However, the resulting interactions may be non-local, and obstructions might even appear if one insists on their locality. The analysis of these obstructions can be carried out by means of standard cohomological techniques.

4 Consistent interactions between the spin-two field and the massive Rarita–Schwinger field

4.1 Standard material: $H(\gamma)$ and $H(\delta|d)$

This section is devoted to the investigation of consistent cross-couplings that can be introduced between a spin-two field and a massive Rarita–Schwinger field. This matter is addressed in the context of the antifield-BRST deformation procedure briefly addressed in the above and relies on computing the solutions to (24)–(26), etc., with the help of the free BRST cohomology.

For obvious reasons, we consider only smooth, local, (background) Lorentz invariant quantities and, moreover, Poincaré invariant quantities (i.e. we do not allow explicit dependence on the spacetime coordinates). The smoothness of the deformations refers to the fact that the deformed solution to the master equation (21) is smooth in the coupling constant λ and reduces to the original solution (20) in the free limit $\lambda = 0$. In addition, we require conservation of the number of derivatives on each field (this condition is frequently met in the literature [14, 16]). If we make the notation $S_1 = \int d^4x\, a$ with a a local function, then (24), which as we have seen controls the first-order deformation, takes the local form

$$sa = \partial_\mu m^\mu\,, \quad \mathrm{gh}\,(a) = 0\,, \quad \varepsilon\,(a) = 0\,, \tag{27}$$

for some local m^μ, and it shows that the non-integrated density of the first-order deformation pertains to the local cohomology of the BRST differential in ghost number zero, $a \in H^0\,(s|d)$, where d denotes the exterior spacetime differential. The solution to (27) is unique up to s-exact pieces plus divergences

$$\begin{aligned} &a \to a + sb + \partial_\mu n^\mu\,, \quad \mathrm{gh}\,(b) = -1\,, \varepsilon\,(b) = 1\,, \\ &\mathrm{gh}\,(n^\mu) = 0\,, \quad \varepsilon\,(n^\mu) = 0\,. \end{aligned} \tag{28}$$

At the same time, if the general solution of (27) is found to be completely trivial, $a = sb + \partial_\mu n^\mu$, then it can be made to vanish $a = 0$.

In order to analyze (27), we develop a according to the antighost number

$$a = \sum_{i=0}^{I} a_i\,, \quad \mathrm{agh}\,(a_i) = i\,, \quad \mathrm{gh}\,(a_i) = 0\,, \quad \varepsilon\,(a_i) = 0\,, \tag{29}$$

and take this decomposition to stop at some finite value I of the antighost number. The fact that I in (29) is finite can be argued like in [16]. Inserting the above expansion into (27) and projecting it on the various values of the antighost number with the help of the split (8), we obtain the tower of equations

$$\gamma a_I = \partial_\mu \overset{(I)}{m}{}^\mu\,, \tag{30}$$

$$\delta a_I + \gamma a_{I-1} = \partial_\mu \overset{(I-1)}{m}{}^\mu\,, \tag{31}$$

$$\delta a_i + \gamma a_{i-1} = \partial_\mu \overset{(i-1)}{m}{}^\mu\,, \quad 1 \le i \le I-1\,, \tag{32}$$

where $\left(\overset{(i)}{m}{}^\mu\right)_{i=\overline{0,I}}$ are some local currents with $\mathrm{agh}\left(\overset{(i)}{m}{}^\mu\right) = i$. Moreover, according to the general result from [16] in the absence of the collection indices, (30) can be replaced[1] in strictly positive antighost numbers by

$$\gamma a_I = 0, \quad I > 0\,. \tag{33}$$

Due to the second-order nilpotency of γ ($\gamma^2 = 0$), the solution to (33) is clearly unique up to γ-exact contributions

$$\begin{aligned} &a_I \to a_I + \gamma b_I\,, \quad \mathrm{agh}\,(b_I) = I\,, \\ &\mathrm{pgh}\,(b_I) = I - 1\,, \quad \varepsilon\,(b_I) = 1\,. \end{aligned} \tag{34}$$

Meanwhile, if it turns out that a_I reduces to γ-exact terms only, $a_I = \gamma b_I$, then it can be made to vanish, $a_I = 0$. The non-triviality of the first-order deformation a is thus translated at its highest antighost number component into the requirement that $a_I \in H^I\,(\gamma)$, where $H^I\,(\gamma)$ denotes the cohomology of the exterior longitudinal derivative γ in pure ghost number equal to I. Thus, in order to solve (27) (equivalent to (33) and (31)–(32)), we need to compute the cohomology of γ, $H\,(\gamma)$, and, as it will be made clear below, also the local cohomology of δ in pure ghost number zero, $H\,(\delta|d)$.

Using the results on the cohomology of the exterior longitudinal differential for a Pauli–Fierz field [16], as well as the definitions (15) and (16), we can state that $H\,(\gamma)$ is generated on the one hand by $\Phi^*_{\alpha_0}$, η^*_μ, ψ_μ and $K_{\mu\nu\alpha\beta}$ together with all of their spacetime derivatives and, on the other hand, by the ghosts η_μ and $\partial_{[\mu}\eta_{\nu]}$. Thus, the most general (and non-trivial), local solution to (33) can be written, up to γ-exact contributions, as

$$a_I = \alpha_I\left([\psi_\mu]\,, [K_{\mu\nu\alpha\beta}]\,, \left[\Phi^*_{\alpha_0}\right], [\eta^*_\mu]\right)\omega^I\left(\eta_\mu, \partial_{[\mu}\eta_{\nu]}\right)\,, \tag{35}$$

where the notation $f\,([q])$ means that f depends on q and its derivatives up to a finite order, while ω^I denotes the elements of a basis in the space of polynomials with pure ghost number I in the corresponding ghosts and their antisymmetrized first-order derivatives. The objects α_I have the pure ghost number equal to zero and are required to fulfill the property $\mathrm{agh}\,(\alpha_I) = I$ in order to ensure that the ghost number of a_I is equal to zero. Since they have a bounded number of derivatives and a finite antighost number, α_I are actually polynomials in the linearized Riemann tensor, in the antifields, in all of their derivatives, as well as in the derivatives of the Rarita–Schwinger fields. The anticommuting behaviour of the vector-spinors induces that α_I are also polynomials in the undifferentiated Rarita–Schwinger fields, so we conclude that these elements exhibit a polynomial character in all of their arguments. Due to their γ-closeness, $\gamma\alpha_I = 0$, α_I will be called invariant polynomials. In zero antighost number the invariant polynomials are polynomials in the linearized Riemann tensor $K_{\mu\nu\alpha\beta}$, in the Rarita–Schwinger spinors, as well as in their derivatives.

Inserting (35) in (31), we obtain that a necessary (but not sufficient) condition for the existence of (non-trivial) solutions a_{I-1} is that the invariant polynomials α_I are (non-trivial) objects from the local cohomology of the Koszul–Tate differential $H\,(\delta|d)$ in pure ghost number zero

[1] This is because the presence of the matter fields does not modify the general results on $H\,(\gamma)$ presented in [16].

and in strictly positive antighost numbers $I>0$

$$\delta\alpha_I = \partial_\mu \overset{(I-1)^\mu}{j}, \quad \mathrm{agh}\left(\overset{(I-1)^\mu}{j}\right) = I-1,$$
$$\mathrm{pgh}\left(\overset{(I-1)^\mu}{j}\right) = 0. \qquad (36)$$

We recall that $H(\delta|d)$ is completely trivial in both strictly positive antighost *and* pure ghost numbers (for instance, see [23], Theorem 5.4 and [26]). Using the fact that the Cauchy order of the free theory under study is equal to two together with the general results from [23], according to which the local cohomology of the Koszul–Tate differential in pure ghost number zero is trivial in antighost numbers strictly greater than its Cauchy order, we can state that

$$H_J(\delta|d) = 0 \quad \text{for all } J>2, \qquad (37)$$

where $H_J(\delta|d)$ represents the local cohomology of the Koszul–Tate differential in zero pure ghost number and in antighost number J. An interesting property of invariant polynomials for the free model under study is that if an invariant polynomial α_J, with $\mathrm{agh}(\alpha_J) = J \geq 2$, is trivial in $H_J(\delta|d)$, then it can be taken to be trivial also in $H_J^{\mathrm{inv}}(\delta|d)$, i.e.

$$\left(\alpha_J = \delta b_{J+1} + \partial_\mu \overset{(J)^\mu}{c}, \ \mathrm{agh}(\alpha_J) = J \geq 2\right)$$
$$\Rightarrow \alpha_J = \delta\beta_{J+1} + \partial_\mu \overset{(J)^\mu}{\gamma}, \qquad (38)$$

with both β_{J+1} and $\overset{(J)^\mu}{\gamma}$ invariant polynomials. Here, $H_J^{\mathrm{inv}}(\delta|d)$ denotes the invariant characteristic cohomology (the local cohomology of the Koszul–Tate differential in the space of invariant polynomials) in antighost number J. This property is proved in [16] in the case of a collection of Pauli–Fierz fields and remains valid in the case considered here, since the matter fields do not carry gauge symmetries. Thus, we can write that

$$H_J^{\mathrm{inv}}(\delta|d) = 0 \quad \text{for all } J>2. \qquad (39)$$

For the same reason, the antifields of the matter fields can bring only trivial contributions to $H_J(\delta|d)$ and $H_J^{\mathrm{inv}}(\delta|d)$ for $J \geq 2$, so the results from [16] concerning both $H_2(\delta|d)$ in pure ghost number zero and $H_2^{\mathrm{inv}}(\delta|d)$ remain valid. These cohomological spaces are still spanned by the undifferentiated antifields corresponding to the ghosts

$$H_2(\delta|d) \text{ and } H_2^{\mathrm{inv}}(\delta|d) : (\eta^{*\mu}). \qquad (40)$$

In contrast to the groups $(H_J(\delta|d))_{J\geq 2}$ and $\left(H_J^{\mathrm{inv}}(\delta|d)\right)_{J\geq 2}$, which are finite-dimensional, the cohomology $H_1(\delta|d)$ in pure ghost number zero, known to be related to global symmetries and ordinary conservation laws, is infinite-dimensional, since the theory is free. Moreover, $H_1(\delta|d)$ non-trivially involves the antifields of the matter fields.

The previous results on $H(\delta|d)$ and $H^{\mathrm{inv}}(\delta|d)$ in strictly positive antighost numbers are important because they control the obstructions to removing the antifields from the first-order deformation. More precisely, based on the formulas (36)–(39), one can successively eliminate all the pieces of antighost number strictly greater that two from the non-integrated density of the first-order deformation by adding only trivial terms. Thus, one can take, without loss of non-trivial objects, the condition $I \leq 2$ in the decomposition (29). In addition, the last representative is of the form (35), where the invariant polynomial is necessarily a non-trivial object from $H_2^{\mathrm{inv}}(\delta|d)$ for $I=2$, and from $H_1(\delta|d)$ for $I=1$, respectively.

4.2 First-order deformation

In the case $I=2$, the non-integrated density of the first-order deformation (29) becomes

$$a = a_0 + a_1 + a_2. \qquad (41)$$

We can further decompose a in a natural manner as a sum between three kinds of deformations

$$a = a^{(\mathrm{PF})} + a^{(\mathrm{int})} + a^{(\mathrm{RS})}, \qquad (42)$$

where $a^{(\mathrm{PF})}$ contains only fields/ghosts/antifields from the Pauli–Fierz sector, $a^{(\mathrm{int})}$ describes the cross-interactions between the two theories (so it effectively mixes both sectors), and $a^{(\mathrm{RS})}$ involves only the Rarita–Schwinger sector. The component $a^{(\mathrm{PF})}$ is completely known (for a detailed analysis see [16]) and satisfies individually an equation of the type (27). It admits a decomposition similar to (41)

$$a^{(\mathrm{PF})} = a_0^{(\mathrm{PF})} + a_1^{(\mathrm{PF})} + a_2^{(\mathrm{PF})}, \qquad (43)$$

where

$$a_2^{(\mathrm{PF})} = \frac{1}{2}\eta^{*\mu}\eta^\nu \partial_{[\mu}\eta_{\nu]}, \qquad (44)$$
$$a_1^{(\mathrm{PF})} = h^{*\mu\rho}\left((\partial_\rho \eta^\nu) h_{\mu\nu} - \eta^\nu \partial_{[\mu} h_{\nu]\rho}\right), \qquad (45)$$

and $a_0^{(\mathrm{PF})}$ is the cubic vertex of the Einstein–Hilbert Lagrangian plus a cosmological term[2]. Due to the fact that $a^{(\mathrm{int})}$ and $a^{(\mathrm{RS})}$ involve different kinds of fields, it follows that $a^{(\mathrm{int})}$ and $a^{(\mathrm{RS})}$ are subject to some separate equations

$$s a^{(\mathrm{int})} = \partial_\mu m^{(\mathrm{int})\mu}, \qquad (46)$$
$$s a^{(\mathrm{RS})} = \partial_\mu m^{(\mathrm{RS})\mu}, \qquad (47)$$

[2] The terms $a_2^{(\mathrm{PF})}$ and $a_1^{(\mathrm{PF})}$ given in (44) and (45) differ from the corresponding ones in [16] by a γ-exact and a δ-exact contribution, respectively. However, the difference between our $a_2^{(\mathrm{PF})} + a_1^{(\mathrm{PF})}$ and the corresponding sum from [16] is an s-exact modulo d quantity. The associated component of antighost number zero, $a_0^{(\mathrm{PF})}$, is nevertheless the same in both formulations. As a consequence, the object $a^{(\mathrm{PF})}$ and the first-order deformation in [16] belong to the same cohomological class from $H^0(s|d)$.

for some local m^{μ}'s. In the following, we analyze the general solutions to these equations.

Since the massive Rarita–Schwinger field does not carry gauge symmetries of its own, the massive gravitino sector can only occur in antighost number one and zero. Thus, without loss of generality, we can take

$$a^{(\mathrm{int})}=a_0^{(\mathrm{int})}+a_1^{(\mathrm{int})} \tag{48}$$

in (46), where the components involved in the right-hand side of (48) are subject to the equations

$$\gamma a_1^{(\mathrm{int})}=0\,, \tag{49}$$

$$\delta a_1^{(\mathrm{int})}+\gamma a_0^{(\mathrm{int})}=\partial_\mu \overset{(0)}{m}{}^{(\mathrm{int})\mu}\,. \tag{50}$$

According to (35) in pure ghost number one and because ω^1 is spanned by

$$\omega^1=\left(\eta_\mu,\partial_{[\mu}\eta_{\nu]}\right)\,,$$

we infer that the most general expression of $a_1^{(\mathrm{int})}$ as solution to (49) is[3]

$$a_1^{(\mathrm{int})}=\psi^{*\mu}\left(N^{\rho}{}_{\mu}\eta_\rho+N^{\rho\lambda}{}_{\mu}\partial_{[\rho}\eta_{\lambda]}\right)\,, \tag{51}$$

where $N^{\rho}{}_{\mu}$ and $N^{\rho\lambda}{}_{\mu}$ are real, odd spinor-like functions, with $N^{\rho\lambda}{}_{\mu}$ antisymmetric in its upper indices. All the objects denoted by N are gauge-invariant, so they may depend on ψ_μ, $K_{\mu\nu\rho\lambda}$, and their spacetime derivatives. At this stage we recall the hypothesis on the conservation of the number of derivatives on each field, which allows us to simplify the solution (51) to (49) by imposing that the following requirements are simultaneously satisfied:

i) the interaction vertices present in $a_0^{(\mathrm{int})}$ as solution to (50), assuming $a_0^{(\mathrm{int})}$ exists, contain at most two derivatives of the fields;

ii) the deformed field equations associated with $a_0^{(\mathrm{int})}$ involve at most the first-order derivatives of the spinor fields and at most the second-order derivatives of the Pauli–Fierz field.

By applying the differential δ on (51) and using the definitions (12)–(16), we infer that

$$\delta a_1^{(\mathrm{int})}=\partial_\mu m^\mu+\gamma b_0+c_0\,, \tag{52}$$

[3] We remark that, in principle, we might have added to $a_1^{(\mathrm{int})}$ a component $\tilde{a}_1^{(\mathrm{int})}$ linear in the antifield of the Pauli–Fierz field, $h^{*\mu\nu}$. However, such terms cannot produce a consistent component of the first-order deformation in antighost number zero, as is shown in Appendix B.

where

$$m^\mu=-\mathrm{i}\bar{\psi}_\beta\gamma^{\mu\beta\nu}\left(N^{\rho}{}_{\nu}\eta_\rho+N^{\rho\lambda}{}_{\mu}\partial_{[\rho}\eta_{\lambda]}\right)\,, \tag{53}$$

$$b_0=\frac{\mathrm{i}}{2}\bar{\psi}_\beta\gamma^{\alpha\beta\mu}\left(N^{\rho}{}_{\mu}h_{\alpha\rho}+2N^{\rho\lambda}{}_{\mu}\partial_{[\rho}h_{\lambda]\alpha}\right)\,, \tag{54}$$

$$\begin{aligned}c_0=&\left(m\bar{\psi}_\alpha\gamma^{\alpha\mu}N^{\rho}{}_{\mu}+\mathrm{i}\bar{\psi}_\beta\gamma^{\alpha\beta\mu}\partial_\alpha N^{\rho}{}_{\mu}\right)\eta_\rho\\&+\Big(m\bar{\psi}_\alpha\gamma^{\alpha\mu}N^{\rho\lambda}{}_{\mu}+\mathrm{i}\bar{\psi}_\beta\gamma^{\alpha\beta\mu}\partial_\alpha N^{\rho\lambda}{}_{\mu}\\&+\frac{\mathrm{i}}{2}\bar{\psi}_\beta\gamma^{\rho\beta\mu}N^{\lambda}{}_{\mu}\Big)\partial_{[\rho}\eta_{\lambda]}\,.\end{aligned} \tag{55}$$

Taking into account the previous two requirements on the derivative behaviour of $a_0^{(\mathrm{int})}$, from (54) we get that the spinor-tensor $N^{\rho}{}_{\mu}$ may contain at most one derivative of the spinor ψ_μ, while the spinor-tensor $N^{\rho\lambda}{}_{\mu}$ can only depend on the undifferentiated Rarita–Schwinger field. As a consequence, we have that

$$N^{\rho}{}_{\mu}=\bar{N}^{\rho\lambda}{}_{\mu}\psi_\lambda+\bar{N}^{\rho\lambda\sigma}{}_{\mu}\partial_\lambda\psi_\sigma,\quad N^{\rho\lambda}{}_{\mu}=N^{\rho\lambda\sigma}{}_{\mu}\psi_\sigma\,, \tag{56}$$

and hence

$$\begin{aligned}a_1^{(\mathrm{int})}=&\,\psi^{*\mu}\left(\bar{N}^{\rho\lambda}{}_{\mu}\psi_\lambda+\bar{N}^{\rho\lambda\sigma}{}_{\mu}\partial_\lambda\psi_\sigma\right)\eta_\rho\\&+\psi^{*\mu}N^{\rho\lambda\sigma}{}_{\mu}\psi_\sigma\partial_{[\rho}\eta_{\lambda]}\,,\end{aligned} \tag{57}$$

where $\bar{N}^{\rho\lambda}{}_{\mu}$, $\bar{N}^{\rho\lambda\sigma}{}_{\mu}$, and $N^{\rho\lambda\sigma}{}_{\mu}$ are real, bosonic 4×4 matrices that may depend only on the undifferentiated spinor-vector ψ_μ. Inserting (56) in (54)–(55), we get

$$\begin{aligned}b_0=&\,\frac{\mathrm{i}}{2}\bar{\psi}_\beta\gamma^{\alpha\beta\mu}\left(\left(\bar{N}^{\rho\lambda}{}_{\mu}\psi_\lambda+\bar{N}^{\rho\lambda\sigma}{}_{\mu}\partial_\lambda\psi_\sigma\right)h_{\alpha\rho}\right.\\&\left.+2N^{\rho\lambda\sigma}{}_{\mu}\psi_\sigma\partial_{[\rho}h_{\lambda]\alpha}\right)\,,\end{aligned} \tag{58}$$

$$\begin{aligned}c_0=&\left(m\bar{\psi}_\alpha\gamma^{\alpha\mu}\left(\bar{N}^{\rho\lambda}{}_{\mu}\psi_\lambda+\bar{N}^{\rho\lambda\sigma}{}_{\mu}\partial_\lambda\psi_\sigma\right)\right.\\&\left.+\mathrm{i}\bar{\psi}_\beta\gamma^{\alpha\beta\mu}\partial_\alpha\left(\bar{N}^{\rho\lambda}{}_{\mu}\psi_\lambda+\bar{N}^{\rho\lambda\sigma}{}_{\mu}\partial_\lambda\psi_\sigma\right)\right)\eta_\rho\\&+\Big(m\bar{\psi}_\alpha\gamma^{\alpha\mu}N^{\rho\lambda\sigma}{}_{\mu}\psi_\sigma+\mathrm{i}\bar{\psi}_\beta\gamma^{\alpha\beta\mu}\partial_\alpha\left(N^{\rho\lambda\sigma}{}_{\mu}\psi_\sigma\right)\\&+\frac{\mathrm{i}}{2}\bar{\psi}_\beta\gamma^{\rho\beta\mu}\left(\bar{N}^{\lambda\sigma}{}_{\mu}\psi_\sigma+\bar{N}^{\lambda\alpha\sigma}{}_{\mu}\partial_\alpha\psi_\sigma\right)\Big)\partial_{[\rho}\eta_{\lambda]}\,.\end{aligned} \tag{59}$$

The condition that $\delta a_1^{(\mathrm{int})}$ should be written like in (50) restricts c_0 expressed in (59) to be a γ-exact modulo d quantity, i.e.

$$c_0=\gamma m+\partial_\mu n^\mu\,. \tag{60}$$

At this stage it is useful to split c_0 as follows:

$$c_0=\sum_{k=0}^{2}\left(c_0\right)_k\,, \tag{61}$$

where $\left(c_0\right)_k$ denotes the piece from c_0 with k-derivatives. According to this decomposition, it follows that each $\left(c_0\right)_k$ should be written in a γ-exact modulo d form, such that (50) is indeed satisfied. Using (59), we obtain that

$$\left(c_0\right)_0=m\bar{\psi}_\alpha\gamma^{\alpha\mu}\bar{N}^{\rho\lambda}{}_{\mu}\psi_\lambda\eta_\rho\,. \tag{62}$$

As the right-hand side of (62) is derivative-free, it follows that these terms neither reduce to a total derivative nor can they be expressed in a γ-exact form, so they must vanish

$$\bar{\psi}_\alpha \gamma^{\alpha\mu} \bar{N}^{\rho\lambda}{}_{\mu} \psi_\lambda = 0 \,. \tag{63}$$

Simple computation exhibits that (63) is checked if

$$\gamma^0 \gamma^{\alpha\mu} \bar{N}^{\rho\lambda}{}_{\mu} = \left(\gamma^0 \gamma^{\lambda\mu} \bar{N}^{\rho\alpha}{}_{\mu}\right)^{\mathsf{T}} , \tag{64}$$

whose general solution is expressed by

$$\bar{N}^{\rho\lambda}{}_{\mu} = c_1 \delta^\rho_\mu \gamma^\lambda + c_2 \delta^\lambda_\mu \gamma^\rho + c_3 \sigma^{\rho\lambda} \gamma_\mu + \frac{1}{2}\left(c_1 + 2c_2 + 3c_3\right) \gamma^{\rho\lambda}{}_{\mu} \,, \tag{65}$$

with c_1, c_2, and c_3 being some arbitrary functions depending on ψ_μ. As is shown in Appendix B, the functions c_1, c_2, and c_3 from (65) can be made to vanish by adding some trivial, s-exact terms and by conveniently redefining the functions $\bar{N}^{\rho\lambda\sigma}{}_{\mu}$. Consequently, we can take

$$\bar{N}^{\rho\lambda}{}_{\mu} = 0 \,. \tag{66}$$

The equation (60) for $k = 1$ becomes

$$m\bar{\psi}_\alpha \gamma^{\alpha\mu} \bar{N}^{\rho\lambda\sigma}{}_{\mu} \left(\partial_\lambda \psi_\sigma\right) \eta_\rho + m\bar{\psi}_\alpha \gamma^{\alpha\mu} N^{\rho\lambda\sigma}{}_{\mu} \psi_\sigma \partial_{[\rho}\eta_{\lambda]} = \gamma m_0 + \partial_\mu n^\mu_0 \,, \tag{67}$$

where $\gamma m_0 = \left(\partial m_0 / \partial h_{\rho\lambda}\right) \partial_{(\rho}\eta_{\lambda)}$. By taking the Euler–Lagrange derivatives of the relation (67) with respect to η_ν, we obtain that the quantity $m\bar{\psi}_\alpha \gamma^{\alpha\mu} \bar{N}^{\rho\lambda\sigma}{}_{\mu} \left(\partial_\lambda \psi_\sigma\right)$ should reduce to a total derivative

$$m\bar{\psi}_\alpha \gamma^{\alpha\mu} \bar{N}^{\rho\lambda\sigma}{}_{\mu} \left(\partial_\lambda \psi_\sigma\right) = \partial_\lambda M^{\rho\lambda} \,. \tag{68}$$

The left-hand side of (68) is a full divergence if the following conditions

$$\partial_\lambda \bar{N}^{\rho\lambda\sigma}{}_{\mu} = 0, \tag{69}$$

$$\gamma^0 \gamma^{\alpha\mu} \bar{N}^{\rho\lambda\sigma}{}_{\mu} = -\left(\gamma^0 \gamma^{\sigma\mu} \bar{N}^{\rho\lambda\alpha}{}_{\mu}\right)^{\mathsf{T}} \tag{70}$$

are simultaneously satisfied. The general solution to (69)–(70) takes the form

$$\begin{aligned}
\bar{N}^{\rho\lambda\sigma}{}_{\mu} &= k_1 \left(\sigma^{\lambda\sigma}\left(\delta^\rho_\mu + \frac{1}{2}\gamma^\rho{}_\mu\right) + \sigma^{\rho\sigma}\left(\delta^\lambda_\mu + \frac{1}{2}\gamma^\lambda{}_\mu\right)\right) \\
&\quad + k_2 \sigma^{\rho\lambda}\left(\delta^\sigma_\mu + \frac{1}{2}\gamma^\sigma{}_\mu\right) + k_4 \sigma^{\rho\lambda}\delta^\sigma_\mu \\
&\quad + k_3 \Big(\sigma^{\lambda\sigma}\delta^\rho_\mu - \sigma^{\rho\sigma}\delta^\lambda_\mu - \delta^\sigma_\mu \gamma^{\rho\lambda} + \gamma^{\rho\lambda\sigma}{}_\mu \\
&\qquad - \frac{1}{2}\left(\delta^\lambda_\mu \gamma^{\rho\sigma} - \delta^\rho_\mu \gamma^{\lambda\sigma}\right) + \frac{1}{2}\left(\sigma^{\sigma\lambda}\gamma^\rho{}_\mu - \sigma^{\rho\sigma}\gamma^\lambda{}_\mu\right)\Big) \\
&= \bar{N}^{\rho\lambda\sigma}_1{}_{\mu} + \bar{N}^{\rho\lambda\sigma}_2{}_{\mu} \,,
\end{aligned} \tag{71}$$

with

$$\bar{N}^{\rho\lambda\sigma}_1{}_{\mu} = k_1 \left(\sigma^{\lambda\sigma}\left(\delta^\rho_\mu + \frac{1}{2}\gamma^\rho{}_\mu\right) + \sigma^{\rho\sigma}\left(\delta^\lambda_\mu + \frac{1}{2}\gamma^\lambda{}_\mu\right)\right) + k_2 \sigma^{\rho\lambda}\left(\delta^\sigma_\mu + \frac{1}{2}\gamma^\sigma{}_\mu\right) + k_4 \sigma^{\rho\lambda}\delta^\sigma_\mu \,, \tag{72}$$

and $(k_i)_{i=\overline{1,4}}$ being some arbitrary constants. Under these circumstances (if (69)–(70) are verified), we find that

$$\begin{aligned}
& m\bar{\psi}_\alpha \gamma^{\alpha\mu} \bar{N}^{\rho\lambda\sigma}{}_{\mu} \left(\partial_\lambda \psi_\sigma\right) \eta_\rho + m\bar{\psi}_\alpha \gamma^{\alpha\mu} N^{\rho\lambda\sigma}{}_{\mu} \psi_\sigma \partial_{[\rho}\eta_{\lambda]} \\
&= \gamma \left(-\frac{1}{4} m\bar{\psi}_\alpha \gamma^{\alpha\mu} \bar{N}^{\rho\lambda\sigma}_1{}_{\mu} \psi_\sigma h_{\rho\lambda}\right) \\
&\quad + \partial_\lambda \left(\frac{1}{2} m\bar{\psi}_\alpha \gamma^{\alpha\mu} \bar{N}^{\rho\lambda\sigma}{}_{\mu} \psi_\sigma \eta_\rho\right) \\
&\quad + m\bar{\psi}_\alpha \gamma^{\alpha\mu} \left(N^{\rho\lambda\sigma}{}_{\mu} + \frac{1}{4}\bar{N}^{\rho\lambda\sigma}_2{}_{\mu}\right) \psi_\sigma \partial_{[\rho}\eta_{\lambda]} \,.
\end{aligned} \tag{73}$$

By comparing the last equation to (67), we observe that the last term from the right-hand side of (73) must be γ-exact modulo d. This takes place if

$$\bar{\psi}_\alpha \gamma^{\alpha\mu} \left(N^{\rho\lambda\sigma}{}_{\mu} + \frac{1}{4}\bar{N}^{\rho\lambda\sigma}_2{}_{\mu}\right) \psi_\sigma = 0, \tag{74}$$

from which we further deduce

$$N^{\rho\lambda\sigma}{}_{\mu} = -\frac{1}{4}\bar{N}^{\rho\lambda\sigma}_2{}_{\mu} + \hat{N}^{\rho\lambda\sigma}{}_{\mu} \,, \tag{75}$$

where $\hat{N}^{\rho\lambda\sigma}{}_{\mu}$ is solution to the equation

$$\bar{\psi}_\alpha \gamma^{\alpha\mu} \hat{N}^{\rho\lambda\sigma}{}_{\mu} \psi_\sigma = 0 \,. \tag{76}$$

It is simple to see that (76) holds if

$$\gamma^0 \gamma^{\alpha\mu} \hat{N}^{\rho\lambda\sigma}{}_{\mu} = \left(\gamma^0 \gamma^{\sigma\mu} \hat{N}^{\rho\lambda\alpha}{}_{\mu}\right)^{\mathsf{T}} , \tag{77}$$

whose general solution is given by

$$\begin{aligned}
\hat{N}^{\rho\lambda\sigma}{}_{\mu} &= \bar{k}_1 \left(\sigma^{\lambda\sigma}\delta^\rho_\mu - \sigma^{\rho\sigma}\delta^\lambda_\mu\right) + \bar{k}_2 \delta^\sigma_\mu \gamma^{\rho\lambda} \\
&\quad + \bar{k}_3 \left(\delta^\lambda_\mu \gamma^{\rho\sigma} - \delta^\rho_\mu \gamma^{\lambda\sigma}\right) + \bar{k}_4 \gamma^{\rho\lambda\sigma}{}_\mu \\
&\quad + \frac{1}{2}\left(\bar{k}_1 - 2\bar{k}_2 + \bar{k}_4\right)\left(\sigma^{\sigma\lambda}\gamma^\rho{}_\mu - \sigma^{\rho\sigma}\gamma^\lambda{}_\mu\right) ,
\end{aligned} \tag{78}$$

with $\left(\bar{k}_i\right)_{i=\overline{1,4}}$ being some arbitrary functions depending on ψ_μ.

Next, we analyze the solution to (60) for $k = 2$. It takes the concrete form

$$\begin{aligned}
& \frac{\mathrm{i}}{2}\bar{\psi}_\beta \left(\gamma^{\alpha\beta\mu}\bar{N}^{\rho\lambda\sigma}{}_{\mu} + \gamma^{\lambda\beta\mu}\bar{N}^{\rho\alpha\sigma}{}_{\mu}\right)\left(\partial_\alpha \partial_\lambda \psi_\sigma\right)\eta_\rho \\
&\quad + \mathrm{i}\bar{\psi}_\beta \gamma^{\alpha\beta\mu}\partial_\alpha \left(N^{\rho\lambda\sigma}{}_{\mu}\psi_\sigma\right)\partial_{[\rho}\eta_{\lambda]} \\
&\quad + \frac{\mathrm{i}}{2}\bar{\psi}_\beta \gamma^{\rho\beta\mu}\bar{N}^{\lambda\alpha\sigma}{}_{\mu}\left(\partial_\alpha \psi_\sigma\right)\partial_{[\rho}\eta_{\lambda]} \\
&= \gamma m_1 + \partial_\mu n^\mu_1 \,,
\end{aligned} \tag{79}$$

with $\bar{N}^{\rho\lambda\sigma}{}_{\mu}$ and $N^{\rho\lambda\sigma}{}_{\mu}$ determined previously. By taking the Euler–Lagrange derivatives of (79) with respect to η_ν and by using the result that $\gamma m_1 = \left(\delta m_1 / \delta h_{\rho\lambda}\right)\partial_{(\rho}\eta_{\lambda)} + \partial_\lambda v^\lambda$, with $\delta m_1 / \delta h_{\rho\lambda}$ being the variational derivative of m_1 with respect to $h_{\rho\lambda}$, it follows that

$$\frac{\mathrm{i}}{2}\bar{\psi}_\beta \left(\gamma^{\alpha\beta\mu}\bar{N}^{\rho\lambda\sigma}{}_{\mu} + \gamma^{\lambda\beta\mu}\bar{N}^{\rho\alpha\sigma}{}_{\mu}\right)\left(\partial_\alpha \partial_\lambda \psi_\sigma\right) = \partial_\lambda P^{\rho\lambda} \,, \tag{80}$$

for some $P^{\rho\lambda}$. The left-hand side of the last equation is written as a full divergence if

$$\left(\partial_\lambda\bar{\psi}_\beta\right)\left(\gamma^{\alpha\beta\mu}\bar{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}+\gamma^{\lambda\beta\mu}\bar{N}^{\rho\alpha\sigma}_{\ \ \ \ \mu}\right)\left(\partial_\alpha\psi_\sigma\right)=0\,, \tag{81}$$

which further produces

$$k_1=k_2=k_3=0\,, \tag{82}$$

such that we have

$$\begin{aligned}&\frac{\mathrm{i}}{2}\bar{\psi}_\beta\left(\gamma^{\alpha\beta\mu}\bar{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}+\gamma^{\lambda\beta\mu}\bar{N}^{\rho\alpha\sigma}_{\ \ \ \ \mu}\right)\left(\partial_\alpha\partial_\lambda\psi_\sigma\right)\eta_\rho\\&=-\frac{\mathrm{i}k_4}{4}\gamma\left(\bar{\psi}_\beta\left(\gamma^{\alpha\beta\sigma}\left(\partial_\alpha\psi_\sigma\right)h+\gamma^{\lambda\beta\sigma}\left(\partial^\rho\psi_\sigma\right)h_{\lambda\rho}\right)\right)\\&\quad+\frac{\mathrm{i}k_4}{8}\bar{\psi}_\beta\left(\sigma^{\alpha\rho}\gamma^{\lambda\beta\sigma}-\sigma^{\alpha\lambda}\gamma^{\rho\beta\sigma}\right)\left(\partial_\alpha\psi_\sigma\right)\partial_{[\rho}\eta_{\lambda]}+\partial_\lambda u^\lambda\,.\end{aligned} \tag{83}$$

On the other hand, it is easy to see that

$$\begin{aligned}&\mathrm{i}\bar{\psi}_\beta\gamma^{\alpha\beta\mu}\partial_\alpha\left(N^{\rho\lambda\sigma}_{\ \ \ \ \mu}\psi_\sigma\right)\partial_{[\rho}\eta_{\lambda]}\\&=-\gamma\left(\mathrm{i}\bar{\psi}_\beta\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\psi_\sigma\partial_{[\rho}h_{\lambda]\alpha}\right)+\partial_\lambda\bar{u}^\lambda\\&\quad-\mathrm{i}\left(\partial_\alpha\bar{\psi}_\beta\right)\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\psi_\sigma\partial_{[\rho}\eta_{\lambda]}\,.\end{aligned} \tag{84}$$

Inserting (83)–(84) in (79) and taking into account the result (82), (79) reduces to

$$\begin{aligned}&-\mathrm{i}\left(\partial_\alpha\bar{\psi}_\beta\right)\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\psi_\sigma\partial_{[\rho}\eta_{\lambda]}\\&-\frac{\mathrm{i}k_4}{8}\bar{\psi}_\beta\left(\sigma^{\alpha\rho}\gamma^{\lambda\beta\sigma}-\sigma^{\alpha\lambda}\gamma^{\rho\beta\sigma}\right)\left(\partial_\alpha\psi_\sigma\right)\partial_{[\rho}\eta_{\lambda]}\\&=\gamma\bar{m}_1+\partial_\mu\bar{n}_1^\mu\,.\end{aligned} \tag{85}$$

Now, we decompose $\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}$ as follows:

$$\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}=\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_1+\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_2\,, \tag{86}$$

with

$$\begin{aligned}\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_1&=\frac{1}{2}\left(\frac{1}{2}\bar{k}_1+\bar{k}_2-2\bar{k}_3-\frac{1}{2}\bar{k}_4\right)\\&\times\left(\sigma^{\lambda\sigma}\gamma^{\alpha\beta\rho}+\sigma^{\lambda\beta}\gamma^{\alpha\sigma\rho}-\sigma^{\rho\sigma}\gamma^{\alpha\beta\lambda}\right.\\&\left.-\sigma^{\rho\beta}\gamma^{\alpha\sigma\lambda}\right)\\&+\bar{k}_3\left(2\sigma^{\sigma\beta}\gamma^{\rho\alpha\lambda}-\sigma^{\sigma\alpha}\gamma^{\rho\beta\lambda}-\sigma^{\beta\alpha}\gamma^{\rho\sigma\lambda}\right)\\&+\left(\bar{k}_1-\bar{k}_2+\bar{k}_3+\bar{k}_4\right)\left(\sigma^{\sigma\rho}\sigma^{\lambda\beta}-\sigma^{\sigma\lambda}\sigma^{\rho\beta}\right)\\&\times\gamma^\alpha+\frac{1}{2}\left(\bar{k}_1+\bar{k}_3\right)\left(\left(\sigma^{\sigma\lambda}\sigma^{\alpha\rho}-\sigma^{\sigma\rho}\sigma^{\lambda\alpha}\right)\right.\\&\left.\times\gamma^\beta+\left(\sigma^{\beta\rho}\sigma^{\alpha\lambda}-\sigma^{\beta\lambda}\sigma^{\alpha\rho}\right)\gamma^\sigma\right)\\&+\frac{1}{2}\left(\bar{k}_3+\bar{k}_4\right)\left(\left(\sigma^{\beta\rho}\sigma^{\alpha\sigma}\right.\right.\\&\left.\left.-\sigma^{\sigma\rho}\sigma^{\alpha\beta}\right)\gamma^\lambda+\left(\sigma^{\sigma\lambda}\sigma^{\alpha\beta}-\sigma^{\beta\lambda}\sigma^{\alpha\sigma}\right)\gamma^\rho\right),\end{aligned} \tag{87}$$

$$\begin{aligned}\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_2&=\frac{1}{2}\left(\frac{1}{2}\bar{k}_1-\bar{k}_2-\frac{1}{2}\bar{k}_4\right)\\&\times\left(\sigma^{\lambda\sigma}\gamma^{\alpha\beta\rho}-\sigma^{\lambda\beta}\gamma^{\alpha\sigma\rho}-\sigma^{\rho\sigma}\gamma^{\alpha\beta\lambda}\right.\\&\left.+\sigma^{\rho\beta}\gamma^{\alpha\sigma\lambda}\right)+\left(\bar{k}_2-\bar{k}_3\right)\\&\times\left(\sigma^{\rho\alpha}\gamma^{\beta\sigma\lambda}-\sigma^{\lambda\alpha}\gamma^{\beta\sigma\rho}\right)\\&+\bar{k}_3\left(\sigma^{\beta\alpha}\gamma^{\rho\sigma\lambda}-\sigma^{\sigma\alpha}\gamma^{\rho\beta\lambda}\right)\\&+\frac{1}{2}\left(\bar{k}_1-2\bar{k}_2+\bar{k}_3+2\bar{k}_4\right)\left(\left(\sigma^{\sigma\lambda}\sigma^{\alpha\rho}\right.\right.\\&\left.-\sigma^{\sigma\rho}\sigma^{\lambda\alpha}\right)\gamma^\beta+\left.\left(\sigma^{\beta\lambda}\sigma^{\alpha\rho}-\sigma^{\beta\rho}\sigma^{\alpha\lambda}\right)\gamma^\sigma\right)\\&+\frac{1}{2}\left(\bar{k}_3+\bar{k}_4\right)\left(2\sigma^{\beta\sigma}\left(\sigma^{\alpha\lambda}\gamma^\rho-\sigma^{\alpha\rho}\gamma^\lambda\right)\right.\\&+\left(\sigma^{\beta\rho}\sigma^{\alpha\sigma}+\sigma^{\sigma\rho}\sigma^{\alpha\beta}\right)\gamma^\lambda\\&\left.-\left(\sigma^{\alpha\sigma}\sigma^{\beta\lambda}+\sigma^{\alpha\beta}\sigma^{\sigma\lambda}\right)\gamma^\rho\right).\end{aligned} \tag{88}$$

By direct computation it can be shown that the two components of $\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}$ satisfy the properties

$$\gamma^0\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_1=-\left(\gamma^0\left(\gamma^{\alpha\sigma\mu}\hat{N}^{\rho\lambda\beta}_{\ \ \ \ \mu}\right)_1\right)^{\mathsf{T}}\,, \tag{89}$$

$$\gamma^0\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_2=\left(\gamma^0\left(\gamma^{\alpha\sigma\mu}\hat{N}^{\rho\lambda\beta}_{\ \ \ \ \mu}\right)_2\right)^{\mathsf{T}}\,. \tag{90}$$

By means of the formulas (89)–(90) we can write

$$\begin{aligned}&-\mathrm{i}\left(\partial_\alpha\bar{\psi}_\beta\right)\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\psi_\sigma\partial_{[\rho}\eta_{\lambda]}\\&=\gamma\left(\frac{\mathrm{i}}{2}\bar{\psi}_\beta\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_1\psi_\sigma\partial_{[\rho}h_{\lambda]\alpha}\right)\\&+\mathrm{i}\bar{\psi}_\beta\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_2\left(\partial_\alpha\psi_\sigma\right)\partial_{[\rho}\eta_{\lambda]}\\&+\partial_\alpha\left(-\frac{\mathrm{i}}{2}\bar{\psi}_\beta\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_1\psi_\sigma\partial_{[\rho}\eta_{\lambda]}\right),\end{aligned} \tag{91}$$

such that

$$\begin{aligned}&-\mathrm{i}\left(\partial_\alpha\bar{\psi}_\beta\right)\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\psi_\sigma\partial_{[\rho}\eta_{\lambda]}\\&-\frac{\mathrm{i}k_4}{8}\bar{\psi}_\beta\left(\gamma^{\lambda\beta\sigma}\sigma^{\alpha\rho}-\gamma^{\rho\beta\sigma}\sigma^{\alpha\lambda}\right)\left(\partial_\alpha\psi_\sigma\right)\partial_{[\rho}\eta_{\lambda]}\\&=\gamma\left(\frac{\mathrm{i}}{2}\bar{\psi}_\beta\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_1\psi_\sigma\partial_{[\rho}h_{\lambda]\alpha}\right)\\&\quad+\partial_\alpha\left(-\frac{\mathrm{i}}{2}\bar{\psi}_\beta\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_1\psi_\sigma\partial_{[\rho}\eta_{\lambda]}\right)\\&\quad-\mathrm{i}\bar{\psi}_\beta\left(\frac{k_4}{8}\left(\sigma^{\alpha\rho}\gamma^{\lambda\beta\sigma}-\sigma^{\alpha\lambda}\gamma^{\rho\beta\sigma}\right)\right.\\&\quad\left.-\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_2\right)\left(\partial_\alpha\psi_\sigma\right)\partial_{[\rho}\eta_{\lambda]}\,.\end{aligned} \tag{92}$$

A comparison of (92) with (85) results in that the last term in (92) has to be γ-exact modulo d. This holds if

$$\begin{aligned}&\mathrm{i}\bar{\psi}_\beta\left(\left(\frac{k_4}{8}\left(\sigma^{\alpha\rho}\gamma^{\lambda\beta\sigma}-\sigma^{\alpha\lambda}\gamma^{\rho\beta\sigma}\right)\right.\right.\\&\left.\left.-\left(\gamma^{\alpha\beta\mu}\hat{N}^{\rho\lambda\sigma}_{\ \ \ \ \mu}\right)_2\right)\left(\partial_\alpha\psi_\sigma\right)\right)=\partial_\alpha\theta^\alpha\end{aligned} \tag{93}$$

for some θ^{α} or, in other words, if

$$M^{\alpha\beta\rho\lambda\sigma} = \gamma^0 \left(\frac{k_4}{8} \left(\sigma^{\alpha\rho}\gamma^{\lambda\beta\sigma} - \sigma^{\alpha\lambda}\gamma^{\rho\beta\sigma} \right) - \left(\gamma^{\alpha\beta\mu} \hat{N}^{\rho\lambda\sigma}_{\quad\ \mu} \right)_2 \right) \tag{94}$$

fulfills the condition

$$M^{\alpha\beta\rho\lambda\sigma} = -\left(M^{\alpha\sigma\rho\lambda\beta} \right)^{\mathsf{T}} . \tag{95}$$

With the help of (90) we obtain the relations

$$M^{\alpha\beta\rho\lambda\sigma} = \left(M^{\alpha\sigma\rho\lambda\beta} \right)^{\mathsf{T}} , \tag{96}$$

which indicate that (95) cannot be satisfied, and hence neither can (93). As a consequence, the term $-\mathrm{i}\bar{\psi}_{\beta} M^{\alpha\beta\rho\lambda\sigma} \left(\partial_{\alpha}\psi_{\sigma}\right) \partial_{[\rho}\eta_{\lambda]}$ from (92) must be canceled, which implies

$$M^{\alpha\beta\rho\lambda\sigma} = 0 . \tag{97}$$

The solution to the above equation reads as

$$\bar{k}_1 = \frac{1}{4} k_4 , \quad \bar{k}_2 = \frac{1}{8} k_4 , \quad \bar{k}_3 = 0 , \quad \bar{k}_4 = 0 . \tag{98}$$

Redenoting k_4 by k, we finally find the relations

$$\begin{aligned} \bar{N}^{\rho\lambda\sigma}_{\quad\ \mu} &= k\sigma^{\rho\lambda}\delta^{\sigma}_{\mu} , \ N^{\rho\lambda\sigma}_{\quad\ \mu} = \hat{N}^{\rho\lambda\sigma}_{\quad\ \mu} \\ &= \frac{1}{4} k \left(\sigma^{\lambda\sigma}\delta^{\rho}_{\mu} - \sigma^{\rho\sigma}\delta^{\lambda}_{\mu} + \frac{1}{2}\delta^{\sigma}_{\mu}\gamma^{\rho\lambda} \right) . \end{aligned} \tag{99}$$

Replacing (66) and (99) in (57), we obtain that

$$\begin{aligned} a_1^{(\mathrm{int})} &= k\psi^{*\mu} \left(\partial^{\nu}\psi_{\mu}\right) \eta_{\nu} + \frac{k}{2}\psi^{*\mu}\psi^{\nu}\partial_{[\mu}\eta_{\nu]} \\ &\quad + \frac{k}{8}\psi^{*\rho}\gamma^{\mu\nu}\psi_{\rho}\partial_{[\mu}\eta_{\nu]} . \end{aligned} \tag{100}$$

Meanwhile, if we insert (99) in (58), (73), (83)–(84), and (92) and the resulting expressions in (52), we deduce that the component of antighost number zero from the first-order deformation is given by

$$\begin{aligned} a_0^{(\mathrm{int})} &= \frac{k}{2} \left(\sigma^{\rho\lambda}\mathcal{L}_0^{(\mathrm{RS})} - \frac{\mathrm{i}}{2}\bar{\psi}_{\mu}\gamma^{\mu\nu\rho}\partial^{\lambda}\psi_{\nu} \right) h_{\rho\lambda} \\ &\quad + \frac{\mathrm{i}k}{4} \left(\frac{1}{2}\bar{\psi}^{\mu}\gamma^{\rho}\psi^{\nu} + \sigma^{\mu\rho}\bar{\psi}^{\nu}\gamma^{\sigma}\psi_{\sigma} + \bar{\psi}_{\sigma}\gamma^{\sigma\rho\mu}\psi^{\nu} \right) \\ &\quad \times \partial_{[\mu}h_{\nu]\rho} + \bar{a}_0^{(\mathrm{int})} , \end{aligned} \tag{101}$$

where $\bar{a}_0^{(\mathrm{int})}$ represents the general, local solution to the homogeneous equation

$$\gamma\bar{a}_0^{(\mathrm{int})} = \partial_{\mu}\bar{m}^{(\mathrm{int})\mu} , \tag{102}$$

with some local $\bar{m}^{(\mathrm{int})\mu}$.

Such solutions correspond to $\bar{a}_1^{(\mathrm{int})} = 0$ and thus they cannot deform either the gauge algebra or the gauge transformations, but simply the Lagrangian at order one in the coupling constant. There are two main types of solutions to (102). The first one corresponds to $\bar{m}^{(\mathrm{int})\mu} = 0$ and is given by gauge-invariant, non-integrated densities constructed from the original fields and their spacetime derivatives. According to (35) for both pure ghost and antighost numbers equal to zero, they are given by $\bar{a}_0'^{(\mathrm{int})} = \bar{a}_0'^{(\mathrm{int})} \left([\psi_{\mu}] , [K_{\mu\nu\alpha\beta}] \right)$, up to the conditions that they effectively describe cross-couplings between the two types of fields and cannot be written in a divergence-like form. Unfortunately, this type of solution must depend on the linearized Riemann tensor (and possibly on its derivatives) in order to provide cross-couplings, and thus would lead to terms with at least two derivatives of the Rarita–Schwinger spinors in the deformed field equations. Thus, by virtue of the derivative order assumption, they must be discarded by setting $\bar{a}_0'^{(\mathrm{int})} = 0$. The second kind of solution is associated with $\bar{m}^{(\mathrm{int})\mu} \neq 0$ in (102) and will be approached below.

We split the solution to (102) for $\bar{m}^{(\mathrm{int})\mu} \neq 0$ along the number of derivatives present in the interaction vertices

$$\bar{a}_0^{(\mathrm{int})} = \sum_{i=0}^{2} \overset{(i)}{\omega} , \tag{103}$$

where $\overset{(i)}{\omega}$ contains i derivatives of the fields. The decomposition (103) yields a similar splitting with respect to (102), which becomes equivalent to three independent equations

$$\gamma\overset{(i)}{\omega} = \partial^{\mu}\overset{(i)}{m}_{\mu} , \quad i = \overline{0,2} . \tag{104}$$

Let us solve (104) for $i = 0$. With the help of the definitions of γ acting on the generators from the BRST complex we obtain

$$\gamma\overset{(0)}{\omega} = -2 \left(\partial_{\nu} \frac{\partial\overset{(0)}{\omega}}{\partial h_{\mu\nu}} \right) \eta_{\mu} + \partial_{\mu}\pi^{\mu} . \tag{105}$$

Thus, $\overset{(0)}{\omega}$ is the solution to (104) for $i = 0$ if and only if

$$\partial_{\nu} \frac{\partial\overset{(0)}{\omega}}{\partial h_{\mu\nu}} = 0 . \tag{106}$$

Since $\overset{(0)}{\omega}$ has no derivatives, (106) implies that $\partial\overset{(0)}{\omega}/\partial h_{\rho\mu}$ must be constant. As the only constant and symmetric tensor in four spacetime dimensions is the flat metric, we can write

$$\frac{\partial\overset{(0)}{\omega}}{\partial h_{\mu\nu}} = p\sigma^{\mu\nu} , \tag{107}$$

with p being a real constant. Integrating (107) results in that the solution to (104) for $i = 0$ reads as

$$\overset{(0)}{\omega} = ph + F\left(\psi_{\mu}\right) ,$$

but since it provides no cross-interactions, we can take

$$\overset{(0)}{\omega} = 0 . \tag{108}$$

Next, we pass to (104) for $i=1$. We obtain that

$$\gamma \overset{(1)}{\omega} = -2\left(\partial_\nu \frac{\delta \overset{(1)}{\omega}}{\delta h_{\mu\nu}}\right)\eta_\mu + \partial_\mu \beta^\mu \,, \tag{109}$$

so $\overset{(1)}{\omega}$ checks (104) for $i=1$ is and only if

$$\partial_\nu \frac{\delta \overset{(1)}{\omega}}{\delta h_{\mu\nu}} = 0\,. \tag{110}$$

Because $\overset{(1)}{\omega}$ includes just one spacetime derivative, the solution to (110) is

$$\frac{\delta \overset{(1)}{\omega}}{\delta h_{\mu\nu}} = \partial_\rho D^{\rho\mu\nu}\,, \tag{111}$$

where $D^{\rho\mu\nu}$ depends only on the undifferentiated fields and is antisymmetric in its first two indices

$$D^{\rho\mu\nu} = -D^{\mu\rho\nu}\,. \tag{112}$$

Since $D^{\rho\mu\nu}$ is derivative-free and $h_{\mu\nu}$ is symmetric, (111) implies that $D^{\rho\mu\nu}$ must be symmetric in its last two indices

$$D^{\rho\mu\nu} = D^{\rho\nu\mu}\,. \tag{113}$$

The properties (112) and (113) further lead to

$$\begin{aligned} D^{\rho\mu\nu} &= -D^{\mu\rho\nu} = -D^{\mu\nu\rho} = D^{\nu\mu\rho} \\ &= D^{\nu\rho\mu} = -D^{\rho\nu\mu} = -D^{\rho\mu\nu}\,, \end{aligned} \tag{114}$$

so $D^{\rho\mu\nu}=0$. Consequently, (111) reduces to

$$\frac{\delta \overset{(1)}{\omega}}{\delta h_{\mu\nu}} = 0\,, \tag{115}$$

whose solution is expressed by

$$\overset{(1)}{\omega} = L\left([\psi_\mu]\right) + \partial_\mu G^\mu\left(\psi_\mu\,, h_{\alpha\beta}\right) \tag{116}$$

and is not suitable as the first term provides no cross-interactions, while the second is trivial, so we have that

$$\overset{(1)}{\omega} = 0\,. \tag{117}$$

In the end, we solve (104) for $i=2$. From the relation

$$\gamma \overset{(2)}{\omega} = -2\left(\partial_\nu \frac{\delta \overset{(2)}{\omega}}{\delta h_{\mu\nu}}\right)\eta_\mu + \partial_\mu \xi^\mu \,, \tag{118}$$

we observe that $\overset{(2)}{\omega}$ verifies (104) for $i=2$ if and only if

$$\partial_\nu \frac{\delta \overset{(2)}{\omega}}{\delta h_{\mu\nu}} = 0\,. \tag{119}$$

The solution to the last equation reads as

$$\frac{\delta \overset{(2)}{\omega}}{\delta h_{\mu\nu}} = \partial_\alpha \partial_\beta U^{\mu\alpha\nu\beta}\,, \tag{120}$$

where $U^{\mu\alpha\nu\beta}$ displays the symmetry properties of the Riemann tensor and involves only the undifferentiated fields ψ_μ and $h_{\mu\nu}$. At this stage it is useful to introduce a derivation in the algebra of the fields $h_{\mu\nu}$ and of their derivatives that counts the powers of the fields and their derivatives, defined by

$$N = \sum_{k\geq 0}\left(\partial_{\mu_1\cdots\mu_k} h_{\mu\nu}\right)\frac{\partial}{\partial\left(\partial_{\mu_1\cdots\mu_k} h_{\mu\nu}\right)}\,. \tag{121}$$

Then, it is easy to see that for every non-integrated density χ, we have that

$$N\chi = h_{\mu\nu}\frac{\delta\chi}{\delta h_{\mu\nu}} + \partial_\mu s^\mu\,. \tag{122}$$

If $\chi^{(l)}$ is a homogeneous polynomial of order $l>0$ in the fields and their derivatives, then $N\chi^{(l)} = l\chi^{(l)}$. Using (120), and (122), we find that

$$N\overset{(2)}{\omega} = -\frac{1}{2}K_{\mu\alpha\nu\beta}U^{\mu\alpha\nu\beta} + \partial_\mu v^\mu\,. \tag{123}$$

We expand $\overset{(2)}{\omega}$ as follows:

$$\overset{(2)}{\omega} = \sum_{l>0} \overset{(2)}{\omega}{}^{(l)}\,, \tag{124}$$

where $N\overset{(2)}{\omega}{}^{(l)} = l\overset{(2)}{\omega}{}^{(l)}$, such that

$$N\overset{(2)}{\omega} = \sum_{l>0} l\overset{(2)}{\omega}{}^{(l)}\,. \tag{125}$$

Comparing (123) with (125), we reach the conclusion that the decomposition (124) induces a similar decomposition with respect to $U^{\mu\alpha\nu\beta}$, i.e.

$$U^{\mu\alpha\nu\beta} = \sum_{l>0} U^{\mu\alpha\nu\beta}_{(l-1)}\,. \tag{126}$$

Substituting (126) into (123) and comparing the resulting expression with (125), we obtain that

$$\overset{(2)}{\omega}{}^{(l)} = -\frac{1}{2l}K_{\mu\alpha\nu\beta}U^{\mu\alpha\nu\beta}_{(l-1)} + \partial_\mu \bar{v}^\mu_{(l)}\,. \tag{127}$$

Introducing (127) in (124), we arrive at

$$\overset{(2)}{\omega} = -\frac{1}{2}K_{\mu\alpha\nu\beta}\bar{U}^{\mu\alpha\nu\beta} + \partial_\mu \bar{v}^\mu\,, \tag{128}$$

where

$$\bar{U}^{\mu\alpha\nu\beta} = \sum_{l>0}\frac{1}{l}U^{\mu\alpha\nu\beta}_{(l-1)}\,. \tag{129}$$

Even if consistent, an $\overset{(2)}{\omega}$ of the type (128) would produce field equations with two spacetime derivatives acting on the Rarita–Schwinger spinors, which breaks the hypothesis on the derivative order of the interacting theory. Thus, we must take

$$\overset{(2)}{\omega} = 0 \,. \tag{130}$$

The results (108), (117), and (130) enable us to take, without loss of generality,

$$\bar{a}_0^{(\mathrm{int})} = 0 \tag{131}$$

in (101).

Finally, we analyze the component $a^{(\mathrm{RS})}$ from (42). As the massive Rarita–Schwinger action from (1) has no non-trivial gauge invariance, it follows that $a^{(\mathrm{RS})}$ can only reduce to its component of antighost number zero

$$a^{(\mathrm{RS})} = a_0^{(\mathrm{RS})}\left([\psi_\mu]\right) \,, \tag{132}$$

which is automatically solution to the equation $sa^{(\mathrm{RS})} \equiv \gamma a_0^{(\mathrm{RS})} = 0$. It comes from $a_1^{(\mathrm{RS})} = 0$ and does not deform the gauge transformations (9), but merely modifies the massive spin-3/2 action. The condition that $a_0^{(\mathrm{RS})}$ is of maximum derivative order equal to one is translated into

$$a_0^{(\mathrm{RS})} = V\left(\psi_\mu\right) + V^{\alpha\beta}\left(\psi_\mu\right)\partial_\alpha\psi_\beta \,, \tag{133}$$

where V and $V^{\alpha\beta}$ are polynomials in the undifferentiated spinor fields (since they anticommute). The first polynomial is a scalar (bosonic and real), while the tensor $V^{\alpha\beta}$ is fermionic and anti-Majorana spinor-like.

The general conclusion of this subsection is that the first-order deformation associated with the Pauli–Field theory plus the massive Rarita–Schwinger field can be written as follows:

$$S_1 = S_1^{(\mathrm{PF})} + S_1^{(\mathrm{int})} \,, \tag{134}$$

with

$$S_1^{(\mathrm{PF})} = \int \mathrm{d}^4x \left(a_0^{(\mathrm{PF})} + a_1^{(\mathrm{PF})} + a_2^{(\mathrm{PF})}\right) \,, \tag{135}$$

and

$$S_1^{(\mathrm{int})} = \int \mathrm{d}^4x \left(a_0^{(\mathrm{int})} + a_1^{(\mathrm{int})} + a_0^{(\mathrm{RS})}\right) \,. \tag{136}$$

The first two components of (136) are expressed by (100) and (101) with $\bar{a}_0^{(\mathrm{int})} = 0$, while $a_0^{(\mathrm{RS})}$ is given by (133). This is the most general form that complies with all the hypotheses that must be satisfied by the deformations, including that related to the derivative order of the deformed Lagrangian.

4.3 Second-order deformation

In this subsection we are interested in determining the complete expression of the second-order deformation for the solution to the master equation, which is known to be subject to (25). Proceeding in the same manner as during the first-order deformation procedure, we can write the second-order deformation of the solution to the master equation like the sum between the Pauli–Fierz and the interacting parts

$$S_2 = S_2^{(\mathrm{PF})} + S_2^{(\mathrm{int})} \,. \tag{137}$$

The piece $S_2^{(\mathrm{PF})}$ describes the second-order deformation in the Pauli–Fierz sector and we will not insist on it, since we are merely interested in the cross-couplings. The term $S_2^{(\mathrm{int})}$ results as the solution to the equation

$$\frac{1}{2}\left(S_1 \,, S_1\right)^{(\mathrm{int})} + sS_2^{(\mathrm{int})} = 0 \,, \tag{138}$$

where

$$\left(S_1 \,, S_1\right)^{(\mathrm{int})} = \left(S_1^{(\mathrm{int})} \,, S_1^{(\mathrm{int})}\right) + 2\left(S_1^{(\mathrm{PF})} \,, S_1^{(\mathrm{int})}\right) \tag{139}$$

and $S_1^{(\mathrm{int})}$ is presented in (136). If we denote by $\Delta^{(\mathrm{int})}$ and $b^{(\mathrm{int})}$ the non-integrated densities of $\left(S_1 \,, S_1\right)^{(\mathrm{int})}$ and of $S_2^{(\mathrm{int})}$, respectively, the local form of (138) becomes

$$\Delta^{(\mathrm{int})} = -2sb^{(\mathrm{int})} + \partial_\mu n^\mu \,, \tag{140}$$

with

$$\mathrm{gh}\left(\Delta^{(\mathrm{int})}\right) = 1, \quad \mathrm{gh}\left(b^{(\mathrm{int})}\right) = 0 \,, \quad \mathrm{gh}\left(n^\mu\right) = 1 \,, \tag{141}$$

for some local current n^μ. Direct computation shows that $\Delta^{(\mathrm{int})}$ decomposes like

$$\Delta^{(\mathrm{int})} = \Delta_0^{(\mathrm{int})} + \Delta_1^{(\mathrm{int})} \,, \quad \mathrm{agh}\left(\Delta_I^{(\mathrm{int})}\right) = I \,, \quad I = 0\,,1 \,, \tag{142}$$

with

$$\begin{aligned}\Delta_1^{(\mathrm{int})} = \gamma\Bigg(&k\Bigg(-\frac{1}{4}\left(\psi^{*[\mu}\psi^{\sigma]} + \frac{1}{2}\psi^{*\rho}\gamma^{\mu\sigma}\psi_\rho\right)\partial_{[\sigma}\eta_{\lambda]}\sigma^{\nu\lambda} \\ &.+ k\psi^{*\sigma}\left(\partial^\mu\psi_\sigma\right)\eta^\nu\Bigg)h_{\mu\nu} \\ &+\frac{k\,(2-k)}{2}\left(\psi^{*\mu}\psi^\nu + \frac{1}{4}\psi^{*\sigma}\gamma^{\mu\nu}\psi_\sigma\right)\eta^\rho\partial_{[\mu}h_{\nu]\rho}\Bigg) \\ &+ k\,(1-k)\Bigg(\psi^{*\mu}\left(\partial^\nu\psi_\mu\right)\eta^\rho\partial_{[\nu}\eta_{\rho]} + \frac{1}{4}\Bigg(\psi^{*[\mu}\psi^{\nu]} \\ &+\frac{1}{2}\psi^{*\sigma}\gamma^{\mu\nu}\psi_\sigma\Bigg)\partial_{[\mu}\eta_{\rho]}\partial_{[\nu}\eta_{\lambda]}\sigma^{\rho\lambda}\Bigg) \,,\end{aligned} \tag{143}$$

and

$$
\begin{aligned}
\Delta_0^{(\mathrm{int})} = \gamma & \left(\frac{k}{4}\mathcal{L}_0^{(\mathrm{RS})} h_{\mu\nu} h^{\mu\nu}\right) \\
& + k\left(-\mathcal{L}_0^{(\mathrm{RS})}\eta^\mu + \frac{\mathrm{i}k}{2}\eta_\sigma \partial^\sigma \left(\bar{\psi}^\mu \gamma^\rho \psi_\rho\right)\right. \\
& + \frac{\mathrm{i}k}{4}\bar{\psi}^\mu \gamma^\rho \psi^\sigma \partial_{[\rho}\eta_{\sigma]} + \frac{\mathrm{i}k}{4}\bar{\psi}_\sigma \gamma^\rho \psi_\rho \partial^{[\mu}\eta^{\sigma]} \\
& \left. + \frac{\mathrm{i}k}{16}\bar{\psi}^\mu \left[\gamma^\rho , \gamma^{\alpha\beta}\right] \psi_\rho \partial_{[\alpha}\eta_{\beta]}\right) \left(\partial^\nu h_{\mu\nu} - \partial_\mu h\right) \\
& + \frac{\mathrm{i}k^2}{4}\left(\eta_\sigma \partial^\sigma \left(\bar{\psi}^\mu \gamma^\alpha \psi^\nu - 2\bar{\psi}_\beta \gamma^{\alpha\beta\mu}\psi^\nu\right)\right. \\
& + \bar{\psi}^\mu \gamma^\alpha \psi_\sigma \partial^{[\nu}\eta^{\sigma]} \\
& - \bar{\psi}_\beta \gamma^{\alpha\beta\mu}\psi_\sigma \partial^{[\nu}\eta^{\sigma]} - \bar{\psi}^\sigma \gamma^{\alpha\beta\mu}\psi^\nu \partial_{[\beta}\eta_{\sigma]} \\
& + \left(\frac{1}{8}\bar{\psi}^\mu \left[\gamma^\alpha , \gamma^{\rho\lambda}\right]\psi^\nu - \frac{1}{4}\bar{\psi}_\beta \left[\gamma^{\alpha\beta\mu} , \gamma^{\rho\lambda}\right]\psi^\nu\right) \\
& \left. \times \partial_{[\rho}\eta_{\lambda]}\right) \partial_{[\mu}h_{\nu]\alpha} \\
& + k^2\left(\eta_\sigma \partial^\sigma \mathcal{L}_0^{(\mathrm{RS})} - \frac{\mathrm{i}}{2}\bar{\psi}_\mu \gamma^{\mu\nu\rho}\left(\partial^\sigma \psi_\rho\right)\partial_\nu \eta_\sigma\right. \\
& + \frac{m}{2}\bar{\psi}_\mu \gamma^{\mu\nu}\psi^\sigma \partial_{[\nu}\eta_{\sigma]} \\
& - \frac{\mathrm{i}}{2}\bar{\psi}_\mu \gamma^{\mu\nu\rho}\partial_\nu \left(\psi^\sigma \partial_{[\rho}\eta_{\sigma]}\right) - \frac{\mathrm{i}}{2}\bar{\psi}^\sigma \gamma^{\mu\nu\rho}\left(\partial_\nu \psi_\rho\right)\partial_{[\mu}\eta_{\sigma]} \\
& + \frac{m}{16}\left(\bar{\psi}_\mu \left[\gamma^{\mu\nu} , \gamma^{\alpha\beta}\right]\psi_\nu\right. \\
& \left. - \mathrm{i}\bar{\psi}_\mu \left[\gamma^{\mu\nu\rho} , \gamma^{\alpha\beta}\right]\partial_\nu \psi_\rho\right)\partial_{[\alpha}\eta_{\beta]} \\
& \left. - \frac{\mathrm{i}}{16}\bar{\psi}_\mu \gamma^{\mu\nu\rho}\gamma^{\alpha\beta}\psi_\rho \partial_\nu \left(\partial_{[\alpha}\eta_{\beta]}\right)\right) h \\
& - \frac{\mathrm{i}k^2}{2}\left(\eta_\sigma \partial^\sigma \left(\bar{\psi}_\mu \gamma^{\mu\nu\rho}\partial^\lambda \psi_\nu\right)\right. \\
& + \bar{\psi}_\mu \gamma^{\mu\nu\rho}\left(\partial^\sigma \psi_\nu\right)\partial^\lambda \eta_\sigma + \frac{1}{2}\bar{\psi}_\mu \gamma^{\mu\nu\rho}\left(\partial^\lambda \psi^\sigma \partial_{[\nu}\eta_{\sigma]}\right) \\
& + \frac{1}{2}\bar{\psi}^\sigma \gamma^{\mu\nu\rho}\left(\partial^\lambda \psi_\nu\right)\partial_{[\mu}\eta_{\sigma]} \\
& + \frac{1}{8}\bar{\psi}_\mu \left[\gamma^{\mu\nu\rho} , \gamma^{\alpha\beta}\right]\left(\partial^\lambda \psi_\nu\right)\partial_{[\alpha}\eta_{\beta]} \\
& \left. + \frac{1}{8}\bar{\psi}_\mu \gamma^{\mu\nu\rho}\gamma^{\alpha\beta}\psi_\nu \partial^\lambda \left(\partial_{[\alpha}\eta_{\beta]}\right)\right) h_{\rho\lambda} \\
& - \frac{\mathrm{i}k}{4}\bar{\psi}_\mu \gamma^{\mu\nu(\rho}\partial^{\lambda)}\psi_\nu \\
& \times \left(h_{\lambda\sigma}\partial_\rho \eta^\sigma - \eta^\sigma \left(\partial_\rho h_{\lambda\sigma} - \partial_\sigma h_{\rho\lambda}\right)\right) + \frac{\mathrm{i}k}{4}\bar{\psi}^\mu \gamma^{(\rho}\psi^{\lambda)} \\
& \times \partial_\mu \left(h_{\lambda\sigma}\partial_\rho \eta^\sigma - \eta^\sigma \left(\partial_\rho h_{\lambda\sigma} - \partial_\sigma h_{\rho\lambda}\right)\right) + \frac{\mathrm{i}k}{4}\bar{\psi}^\mu \gamma^\rho \psi_\rho \\
& \times \partial^\nu \left(h_{\sigma(\mu}\partial_{\nu)}\eta^\sigma - \eta^\sigma \left(\partial_{(\mu}h_{\nu)\sigma} - 2\partial_\sigma h_{\mu\nu}\right)\right) \\
& - \frac{\mathrm{i}k}{2}\bar{\psi}^\mu \gamma^\rho \psi_\rho \\
& \times \partial_\mu \left(h^{\alpha\beta}\partial_\alpha \eta_\beta - \eta^\alpha \left(\partial^\beta h_{\alpha\beta} - \partial_\alpha h\right)\right) \\
& - \frac{\mathrm{i}k}{4}\bar{\psi}_\mu \gamma^{\mu\nu(\rho}\psi^{\lambda)} \times \\
& \times \partial_\nu \left(h_{\lambda\sigma}\partial_\rho \eta^\sigma - \eta^\sigma \left(\partial_\rho h_{\lambda\sigma} - \partial_\sigma h_{\rho\lambda}\right)\right) \\
& + 2k\left(\partial^\mu V + V^{\alpha\beta}\partial_\alpha \psi_\beta\right)\eta_\mu + 2kV^{\mu\nu}\left(\partial^\sigma \psi_\nu\right)\partial_\mu \eta_\sigma \\
& + k\frac{\partial^R V}{\partial \psi_\mu}\psi^\nu \partial_{[\mu}\eta_{\nu]} \\
& + kV^{\mu\nu}\partial_\mu \left(\psi^\sigma \partial_{[\nu}\eta_{\sigma]}\right) + k\bar{\psi}^\sigma \frac{\partial^L V^{\mu\nu}}{\partial \bar{\psi}_\rho}\left(\partial_\mu \psi_\nu\right)\partial_{[\rho}\eta_{\sigma]} \\
& + \frac{k}{4}\left(\frac{\partial^R V}{\partial \psi_\rho}\gamma^{\alpha\beta}\psi^\rho - \bar{\psi}_\rho \gamma^{\alpha\beta}\frac{\partial^L V^{\mu\nu}}{\partial \bar{\psi}_\rho}\partial_\mu \psi_\nu\right)\partial_{[\alpha}\eta_{\beta]} \\
& + \frac{k}{4}V^{\mu\nu}\gamma^{\alpha\beta}\partial_\mu \left(\psi_\nu \partial_{[\alpha}\eta_{\beta]}\right) .
\end{aligned}
\tag{144}
$$

Since the first-order deformation in the interacting sector starts in antighost number one, we can take, without loss of generality, the corresponding second-order deformation to start in antighost number two

$$
\begin{gathered}
b^{(\mathrm{int})} = b_0^{(\mathrm{int})} + b_1^{(\mathrm{int})} + b_2^{(\mathrm{int})} , \\
\mathrm{agh}\left(b_I^{(\mathrm{int})}\right) = I, \quad I = 0, 1, 2,
\end{gathered}
\tag{145}
$$

$$
n^\mu = n_0^\mu + n_1^\mu + n_2^\mu , \quad \mathrm{agh}\left(n_I^\mu\right) = I , \quad I = 0, 1, 2 .
\tag{146}
$$

By projecting (140) on various antighost numbers, we obtain

$$
\gamma b_2^{(\mathrm{int})} = \partial_\mu \left(\frac{1}{2}n_2^\mu\right) ,
\tag{147}
$$

$$
\Delta_1^{(\mathrm{int})} = -2\left(\delta b_2^{(\mathrm{int})} + \gamma b_1^{(\mathrm{int})}\right) + \partial_\mu n_1^\mu ,
\tag{148}
$$

$$
\Delta_0^{(\mathrm{int})} = -2\left(\delta b_1^{(\mathrm{int})} + \gamma b_0^{(\mathrm{int})}\right) + \partial_\mu n_0^\mu .
\tag{149}
$$

Equation (147) can always be replaced, by adding trivial terms, with

$$
\gamma b_2^{(\mathrm{int})} = 0 .
\tag{150}
$$

Looking at $\Delta_1^{(\mathrm{int})}$ given in (143), results in that it can be written like in (148) if

$$
\begin{aligned}
\chi = k(1-k) & \left(\psi^{*\mu}\left(\partial^\nu \psi_\mu\right)\eta^\rho \partial_{[\nu}\eta_{\rho]} + \frac{1}{4}\left(\psi^{*[\mu}\psi^{\nu]}\right.\right. \\
& \left.\left. + \frac{1}{2}\psi^{*\sigma}\gamma^{\mu\nu}\psi_\sigma\right)\partial_{[\mu}\eta_{\rho]}\partial_{[\nu}\eta_{\lambda]}\sigma^{\rho\lambda}\right)
\end{aligned}
\tag{151}
$$

can be expressed like

$$
\chi = \delta\varphi + \gamma\omega + \partial_\alpha l^\alpha .
\tag{152}
$$

Supposing that (152) holds and applying δ on it, we infer that

$$
\delta\chi = \gamma(-\delta\omega) + \partial_\alpha \left(\delta l^\alpha\right) .
\tag{153}
$$

On the other hand, using the concrete expression of χ, we have that

$$\begin{aligned}\delta\chi = {} & \gamma\left(\frac{k(1-k)}{2}\delta\left(\psi^{*\rho}\psi_\rho\eta_\nu\left(\partial_\mu h^{\mu\nu}-\partial^\nu h\right)\right)\right)\\ & +\partial^\mu\left(\frac{1}{2}k(1-k)\delta\left(\psi^{*\rho}\psi_\rho\eta^\nu\partial_{[\mu}\eta_{\nu]}\right)\right)\\ & +\gamma\left(\frac{\mathrm{i}}{4}k(1-k)\left(\left(\bar\psi_\beta\gamma^{\alpha\beta\sigma}\left(\partial^\mu\psi_\sigma\right)h^\rho_\alpha-\left(\bar\psi_\beta\gamma^{\alpha\beta[\mu}\psi^{\nu]}\right.\right.\right.\right.\\ & \left.\left.-\bar\psi^\mu\gamma^\alpha\psi^\nu-\sigma^{\alpha[\mu}\bar\psi^{\nu]}\gamma^\sigma\psi_\sigma\right)\sigma^{\rho\lambda}\partial_{[\nu}h_{\lambda]\alpha}\right)\partial_{[\mu}\eta_{\rho]}\\ & \left.\left.-2\bar\psi_\beta\gamma^{\alpha\beta\mu}\left(\partial^\nu\psi_\mu\right)\eta^\rho\partial_{[\nu}h_{\rho]\alpha}\right)\right)\\ & +\partial_\alpha\left(\frac{\mathrm{i}}{2}k(1-k)\left(\bar\psi_\beta\gamma^{\alpha\beta\sigma}\left(\partial^\mu\psi_\sigma\right)\eta^\rho\right.\right.\\ & -\frac{1}{4}\left(\bar\psi_\beta\gamma^{\alpha\beta[\mu}\psi^{\nu]}-\bar\psi^\mu\gamma^\alpha\psi^\nu-\sigma^{\alpha[\mu}\bar\psi^{\nu]}\gamma^\sigma\psi_\sigma\right)\\ & \left.\left.\times\,\sigma^{\rho\lambda}\partial_{[\nu}\eta_{\lambda]}\right)\partial_{[\mu}\eta_{\rho]}\right). \end{aligned} \tag{154}$$

The right-hand side of (154) can be written like in the right-hand side of (153) if the following conditions are simultaneously satisfied

$$\begin{aligned}\delta\omega' = {} & \left(\bar\psi_\beta\gamma^{\alpha\beta\sigma}\left(\partial^\mu\psi_\sigma\right)h^\rho_\alpha-\left(\bar\psi_\beta\gamma^{\alpha\beta[\mu}\psi^{\nu]}\right.\right.\\ & \left.\left.-\bar\psi^\mu\gamma^\alpha\psi^\nu-\sigma^{\alpha[\mu}\bar\psi^{\nu]}\gamma^\sigma\psi_\sigma\right)\sigma^{\rho\lambda}\partial_{[\nu}h_{\lambda]\alpha}\right)\partial_{[\mu}\eta_{\rho]}\\ & -2\bar\psi_\beta\gamma^{\alpha\beta\mu}\left(\partial^\nu\psi_\mu\right)\eta^\rho\partial_{[\nu}h_{\rho]\alpha}\,, \end{aligned} \tag{155}$$

$$\begin{aligned}\delta l'^\alpha = {} & \left(\bar\psi_\beta\gamma^{\alpha\beta\sigma}\left(\partial^\mu\psi_\sigma\right)\eta^\rho-\frac{1}{4}\left(\bar\psi_\beta\gamma^{\alpha\beta[\mu}\psi^{\nu]}\right.\right.\\ & \left.\left.-\bar\psi^\mu\gamma^\alpha\psi^\nu-\sigma^{\alpha[\mu}\bar\psi^{\nu]}\gamma^\sigma\psi_\sigma\right)\sigma^{\rho\lambda}\partial_{[\nu}\eta_{\lambda]}\right)\partial_{[\mu}\eta_{\rho]}\,. \end{aligned} \tag{156}$$

Since none of the quantities $h_{\mu\beta}$, $\partial^{[\alpha}h^{\beta]\lambda}$, η_β, or $\partial^{[\alpha}\eta^{\beta]}$ are δ-exact, the last relations hold if the equations

$$\bar\psi_\beta\gamma^{\alpha\beta\sigma}\left(\partial_\mu\psi_\sigma\right)=\delta\Omega^\alpha{}_\mu\,, \tag{157}$$

$$\bar\psi_\beta\gamma^{\alpha\beta[\mu}\psi^{\nu]}-\bar\psi^\mu\gamma^\alpha\psi^\nu-\sigma^{\alpha[\mu}\bar\psi^{\nu]}\gamma^\sigma\psi_\sigma=\delta\Gamma^{\mu\nu\alpha} \tag{158}$$

take place simultaneously. Assuming that both (157) and (158) are valid, they further give

$$\partial_\alpha\left(\bar\psi_\beta\gamma^{\alpha\beta\sigma}\left(\partial_\mu\psi_\sigma\right)\right)=\delta\left(\partial_\alpha\Omega^\alpha{}_\mu\right)\,, \tag{159}$$

$$\partial_\alpha\left(\bar\psi_\beta\gamma^{\alpha\beta[\mu}\psi^{\nu]}-\bar\psi^\mu\gamma^\alpha\psi^\nu-\sigma^{\alpha[\mu}\bar\psi^{\nu]}\gamma^\sigma\psi_\sigma\right)=\delta\left(\partial_\alpha\Gamma^{\mu\nu\alpha}\right)\,. \tag{160}$$

On the other hand, by direct computation we obtain that

$$\partial_\alpha\left(\bar\psi_\beta\gamma^{\alpha\beta\sigma}\left(\partial_\mu\psi_\sigma\right)\right)=\delta\left(-\mathrm{i}\left(\psi^{*\sigma}\left(\partial_\mu\psi_\sigma\right)-\bar\psi^\sigma\left(\partial_\mu\bar\psi^*_\sigma\right)\right)\right)\,, \tag{161}$$

$$\begin{aligned} & \partial_\alpha\left(\bar\psi_\beta\gamma^{\alpha\beta[\mu}\psi^{\nu]}-\bar\psi^\mu\gamma^\alpha\psi^\nu-\sigma^{\alpha[\mu}\bar\psi^{\nu]}\gamma^\sigma\psi_\sigma\right)\\ & =\delta\left(-\mathrm{i}\psi^{*\sigma}\gamma^{\mu\nu}\psi_\sigma-2\mathrm{i}\psi^{*[\mu}\psi^{\nu]}\right)-\bar\psi_\alpha\gamma^{\alpha\beta[\mu}\partial^{\nu]}\psi_\beta\,, \end{aligned} \tag{162}$$

so the right-hand sides of (161)–(162) cannot be written like in the right-hand sides of (159)–(160). This means that the relations (157)–(158) are not valid, and therefore neither are (155)–(156). As a consequence, χ must vanish, and hence we must set

$$k\,(1-k)=0\,. \tag{163}$$

Using (163), we conclude that

$$k=1\,. \tag{164}$$

Inserting (164) in (143), we obtain that

$$\begin{aligned}\Delta_1^{(\mathrm{int})} = {} & \gamma\left(\left(-\frac{1}{4}\left(\psi^{*[\mu}\psi^{\sigma]}+\frac{1}{2}\psi^{*\rho}\gamma^{\mu\sigma}\psi_\rho\right)\partial_{[\sigma}\eta_{\lambda]}\sigma^{\nu\lambda}\right.\right.\\ & \left.+\psi^{*\sigma}\left(\partial^\mu\psi_\sigma\right)\eta^\nu\right)h_{\mu\nu}\\ & \left.+\frac{1}{2}\left(\psi^{*\mu}\psi^\nu+\frac{1}{4}\psi^{*\sigma}\gamma^{\mu\nu}\psi_\sigma\right)\eta^\rho\partial_{[\mu}h_{\nu]\rho}\right). \end{aligned} \tag{165}$$

Comparing (165) with (148), we find that

$$b_2^{(\mathrm{int})}=0\,, \tag{166}$$

$$\begin{aligned}b_1^{(\mathrm{int})} = {} & \frac{1}{8}\left(\psi^{*[\mu}\psi^{\sigma]}+\frac{1}{2}\psi^{*\rho}\gamma^{\mu\sigma}\psi_\rho\right)h^\lambda_\mu\partial_{[\sigma}\eta_{\lambda]}\\ & -\frac{1}{2}\psi^{*\sigma}\left(\partial^\mu\psi_\sigma\right)\eta^\nu h_{\mu\nu}\\ & -\frac{1}{4}\left(\psi^{*\mu}\psi^\nu+\frac{1}{4}\psi^{*\sigma}\gamma^{\mu\nu}\psi_\sigma\right)\eta^\rho\partial_{[\mu}h_{\nu]\rho}\,. \end{aligned} \tag{167}$$

Substituting (164) in (144) and using (167), we deduce

$$\begin{aligned}\Delta_0^{(\mathrm{int})}+2\delta b_1^{(\mathrm{int})} = {} & \partial_\mu n_0^\mu+\gamma\left(-\frac{1}{4}\mathcal{L}_0^{(\mathrm{RS})}\left(h^2-2h_{\mu\nu}h^{\mu\nu}\right)\right.\\ & +\frac{\mathrm{i}}{8}\bar\psi^\mu\gamma^\lambda\psi^\nu\left(\left(h^\lambda_\rho-h\delta^\lambda_\rho\right)\partial_{[\mu}h_{\nu]\lambda}\right.\\ & \left.+h^\sigma_\nu\left(2\partial_{[\mu}h_{\sigma]\lambda}+\partial_\lambda h_{\mu\sigma}\right)\right)\\ & +\frac{\mathrm{i}}{4}\bar\psi^\mu\gamma^\sigma\psi_\sigma\left(h\left(\partial_\mu h-\partial^\nu h_{\mu\nu}\right)\right.\\ & +h^\rho_\mu\left(\partial^\lambda h_{\rho\lambda}-\partial_\rho h\right)-2h^{\alpha\beta}\partial_\mu h_{\alpha\beta}\\ & \left.+\frac{3}{2}h^{\rho\lambda}\partial_\rho h_{\mu\lambda}+\frac{1}{2}h_{\mu\nu}\partial_\rho h^{\rho\nu}\right)\\ & +\frac{\mathrm{i}}{4}\bar\psi_\mu\gamma^{\mu\nu\beta}\left(\partial^\alpha\psi_\nu\right)\left(hh_{\alpha\beta}-\frac{3}{2}h_{\alpha\sigma}h^\sigma_\beta\right)\\ & -\left(V+V^{\mu\nu}\partial_\mu\psi_\nu\right)h\\ & +\frac{\mathrm{i}}{8}\bar\psi_\beta\gamma^{\beta\mu\alpha}\psi^\nu\left(\left(h\delta^\rho_\nu-\frac{1}{2}h^\rho_\nu\right)\partial_{[\mu}h_{\alpha]\rho}\right.\\ & \left.+h^\rho_\mu\left(3\partial_\alpha h_{\nu\rho}-2\partial_\rho h_{\alpha\nu}\right)\right)\\ & +V^{\mu\nu}\left(h_{\mu\sigma}\partial^\sigma\psi_\nu+\psi^\sigma\partial_{[\nu}h_{\sigma]\mu}\right.\\ & \left.\left.+\frac{1}{4}\gamma^{\alpha\beta}\psi_\nu\partial_{[\alpha}h_{\beta]\mu}\right)\right)+\Pi^{\mu\nu}\partial_{[\mu}\eta_{\nu]}\,, \end{aligned} \tag{168}$$

where

$$\Pi^{\mu\nu} = V^{\mu\rho}\partial^{\nu}\psi_{\rho} + \frac{\partial^R V}{\partial\psi_{\mu}}\psi^{\nu} + V^{\rho\mu}\partial_{\rho}\psi^{\nu} + \bar{\psi}^{\nu}\frac{\partial^L V^{\rho\lambda}}{\partial\bar{\psi}_{\mu}}\partial_{\rho}\psi_{\lambda} + \frac{1}{4}\left(\frac{\partial^R V}{\partial\psi_{\rho}}\gamma^{\mu\nu}\psi_{\rho} + V^{\rho\lambda}\gamma^{\mu\nu}\partial_{\rho}\psi_{\lambda} - \bar{\psi}_{\theta}\gamma^{\mu\nu}\frac{\partial^L V^{\rho\lambda}}{\partial\bar{\psi}_{\theta}}\partial_{\rho}\psi_{\lambda}\right). \quad (169)$$

We observe that (168) can be written like in (149) if and only if

$$\Pi^{\mu\nu} - \Pi^{\nu\mu} = \partial_{\rho}U^{\rho\mu\nu}. \quad (170)$$

The right-hand side of (169) splits according to the number of derivatives into

$$\Pi^{\mu\nu} = \Pi_0^{\mu\nu} + \Pi_1^{\mu\nu}, \quad (171)$$

where we made the notations

$$\Pi_0^{\mu\nu} = \frac{\partial^R V}{\partial\psi_{\mu}}\psi^{\nu} + \frac{1}{4}\frac{\partial^R V}{\partial\psi_{\rho}}\gamma^{\mu\nu}\psi_{\rho}, \quad (172)$$

$$\Pi_1^{\mu\nu} = V^{\mu\rho}\partial^{\nu}\psi_{\rho} + V^{\rho\mu}\partial_{\rho}\psi^{\nu} + \bar{\psi}^{\nu}\frac{\partial^L V^{\rho\lambda}}{\partial\bar{\psi}_{\mu}}\partial_{\rho}\psi_{\lambda} + \frac{1}{4}\left(V^{\rho\lambda}\gamma^{\mu\nu}\partial_{\rho}\psi_{\lambda} - \bar{\psi}_{\theta}\gamma^{\mu\nu}\frac{\partial^L V^{\rho\lambda}}{\partial\bar{\psi}_{\theta}}\partial_{\rho}\psi_{\lambda}\right). \quad (173)$$

As $\Pi_0^{\mu\nu}$ has no derivatives, it cannot bring a divergence-like contribution to (170), and $\Pi_1^{\mu\nu}$ contains just one derivative, so in principle it may lead to a total derivative, as required by (170). As a consequence, from (170) projected on the number of derivatives equal to zero, we find that $\Pi_0^{\mu\nu}$ is subject to the equation

$$\Pi_0^{\mu\nu} - \Pi_0^{\nu\mu} = 0, \quad (174)$$

which is, via (172), equivalent to

$$\frac{\partial^R V}{\partial\psi_{\mu}}\psi^{\nu} - \frac{\partial^R V}{\partial\psi_{\nu}}\psi^{\mu} = -\frac{1}{2}\frac{\partial^R V}{\partial\psi_{\rho}}\gamma^{\mu\nu}\psi_{\rho}. \quad (175)$$

If we generically represent $\partial^R V/\partial\psi_{\mu}$ in the form

$$\frac{\partial^R V}{\partial\psi_{\mu}} = \bar{\psi}_{\alpha}M^{\alpha\mu}\left(\psi^{\nu}\right), \quad (176)$$

(175) requires that

$$\gamma^0 V^{\mu\nu\alpha\sigma} = \left(\gamma^0 V^{\mu\nu\sigma\alpha}\right)^{\mathsf{T}}, \quad (177)$$

where

$$V^{\mu\nu\alpha\sigma} = M^{\alpha\mu}\sigma^{\nu\sigma} - M^{\alpha\nu}\sigma^{\mu\sigma} + \frac{1}{2}M^{\alpha\sigma}\gamma^{\mu\nu} = -V^{\nu\mu\alpha\sigma}. \quad (178)$$

If we decompose $V^{\mu\nu\alpha\sigma}$ as

$$V^{\mu\nu\alpha\sigma} = V_0^{\mu\nu\alpha\sigma}\mathbf{1} + V_1^{\mu\nu\alpha\sigma}{}_{\tau}\gamma^{\tau} + V_2^{\mu\nu\alpha\sigma}{}_{\tau\gamma}\gamma^{\tau\gamma} + V_3^{\mu\nu\alpha\sigma}{}_{\tau\gamma\rho}\gamma^{\tau\gamma\rho} + V_4^{\mu\nu\alpha\sigma}{}_{\tau\gamma\rho\lambda}\gamma^{\tau\gamma\rho\lambda}, \quad (179)$$

the condition (177) implies the relations

$$V_0^{\mu\nu\alpha\sigma} = -V_0^{\mu\nu\sigma\alpha}, \; V_1^{\mu\nu\alpha\sigma}{}_{\tau} = V_1^{\mu\nu\sigma\alpha}{}_{\tau}, \; V_2^{\mu\nu\alpha\sigma}{}_{\tau\gamma} = V_2^{\mu\nu\sigma\alpha}{}_{\tau\gamma}, \quad (180)$$

$$V_3^{\mu\nu\alpha\sigma}{}_{\tau\gamma\rho} = -V_3^{\mu\nu\sigma\alpha}{}_{\tau\gamma\rho}, \; V_4^{\mu\nu\alpha\sigma}{}_{\tau\gamma\rho\lambda} = -V_4^{\mu\nu\sigma\alpha}{}_{\tau\gamma\rho\lambda}. \quad (181)$$

In a similar manner, if we expand $M^{\alpha\mu}$ along the basis in the space of constant, 4×4 complex matrices

$$M^{\alpha\mu} = M_0^{\alpha\mu}\mathbf{1} + M_1^{\alpha\mu}{}_{\tau}\gamma^{\tau} + M_2^{\alpha\mu}{}_{\tau\gamma}\gamma^{\tau\gamma} + M_3^{\alpha\mu}{}_{\tau\gamma\rho}\gamma^{\tau\gamma\rho} + M_4^{\alpha\mu}{}_{\tau\gamma\rho\lambda}\gamma^{\tau\gamma\rho\lambda}, \quad (182)$$

substitute (182) in (178), and take into account (180)–(181), then we finally find that

$$M_0^{\alpha\mu} = m_0\left(\psi^{\nu}\right)\sigma^{\alpha\mu}, \quad M_1^{\alpha\mu}{}_{\tau} = 0, \quad M_2^{\alpha\mu}{}_{\tau\gamma} = m_2\left(\psi^{\nu}\right)\delta^{\alpha}_{[\tau}\delta^{\mu}_{\gamma]}, \quad (183)$$

$$M_3^{\alpha\mu}{}_{\tau\gamma\rho} = 0, \quad M_4^{\alpha\mu}{}_{\tau\gamma\rho\lambda} = m_4\left(\psi^{\nu}\right)\varepsilon_{\tau\gamma\rho\lambda}\sigma^{\alpha\mu}, \quad (184)$$

where $m_0\left(\psi^{\nu}\right)$, $m_2\left(\psi^{\nu}\right)$,and $m_4\left(\psi^{\nu}\right)$ are arbitrary functions. Replacing now (183)–(184) in (182) and then the resulting expression in (176), we find that

$$\frac{\partial^R V}{\partial\psi_{\mu}} = m_0\left(\psi^{\nu}\right)\bar{\psi}^{\mu} + 2m_2\left(\psi^{\nu}\right)\bar{\psi}_{\alpha}\gamma^{\alpha\mu} + 24\mathrm{i}m_4\left(\psi^{\nu}\right)\bar{\psi}_{\mu}\gamma_5 = \frac{1}{2}m_0\left(\psi^{\nu}\right)\frac{\partial^R X}{\partial\psi_{\mu}} + m_2\left(\psi^{\nu}\right)\frac{\partial^R Y}{\partial\psi_{\mu}} + 12m_4\left(\psi^{\nu}\right)\frac{\partial^R Z}{\partial\psi_{\mu}}, \quad (185)$$

with

$$X \equiv \bar{\psi}_{\mu}\psi^{\mu}, \quad Y \equiv \bar{\psi}_{\alpha}\gamma^{\alpha\mu}\psi_{\mu}, \quad Z \equiv \mathrm{i}\bar{\psi}_{\mu}\gamma_5\psi^{\mu}. \quad (186)$$

The equation (185) shows that the solution to (175) is nothing but an arbitrary polynomial of X, Y, and Z, i.e.

$$V = V\left(X, Y, Z\right). \quad (187)$$

In order to complete the analysis of (170), we need to solve its component of order one in the spacetime derivatives

$$\Pi_1^{\mu\nu} - \Pi_1^{\nu\mu} = \partial_{\rho}U^{\rho\mu\nu}, \quad (188)$$

with $\Pi_1^{\mu\nu}$ given in (173) and $U^{\rho\mu\nu}$ containing no derivatives. Taking into consideration (173), it follows that (188) restricts $V^{\mu\lambda}$ to satisfy the equation

$$V^{\mu\lambda}\sigma^{\nu\rho} + V^{\rho\mu}\sigma^{\nu\lambda} - V^{\nu\lambda}\sigma^{\mu\rho} - V^{\rho\nu}\sigma^{\mu\lambda} + \bar{\psi}^{\nu}\frac{\partial^L V^{\rho\lambda}}{\partial\bar{\psi}_{\mu}} - \bar{\psi}^{\mu}\frac{\partial^L V^{\rho\lambda}}{\partial\bar{\psi}_{\nu}} + \frac{1}{2}\left(V^{\rho\lambda}\gamma^{\mu\nu} - \bar{\psi}_{\theta}\gamma^{\mu\nu}\frac{\partial^L V^{\rho\lambda}}{\partial\bar{\psi}_{\theta}}\right) = \frac{\partial^R U^{\rho\mu\nu}}{\partial\psi_{\lambda}}. \quad (189)$$

The last equation is fulfilled if there exist some objects Q^{μ} such that the following conditions take place simultaneously:

$$V^{\mu\lambda} = -\frac{\partial^R Q^\mu}{\partial \psi_\lambda}, \tag{190}$$

$$\frac{\partial^R Q^\rho}{\partial \psi_\mu}\sigma^{\nu\lambda} - \frac{\partial^R Q^\rho}{\partial \psi_\nu}\sigma^{\mu\lambda} = 0. \tag{191}$$

On the other hand, by adding to and subtracting from the left-hand side of (189) the quantity $(1/2)\left(\partial^R Q^\rho/\partial\psi_\lambda\right) \times\gamma^{\mu\nu} = \partial^R\left(1/2\left(Q^\rho\gamma^{\mu\nu}\right)\right)/\partial\psi_\lambda$, we can state that (189) is checked if (190) and

$$\frac{\partial^R Q^\rho}{\partial \psi_\mu}\sigma^{\nu\lambda} - \frac{\partial^R Q^\rho}{\partial \psi_\nu}\sigma^{\mu\lambda} + \frac{1}{2}\frac{\partial^R Q^\rho}{\partial \psi_\lambda}\gamma^{\mu\nu} = 0 \tag{192}$$

are simultaneously verified. By multiplying (192) from the right with ψ_λ we obtain the equation

$$\frac{\partial^R Q^\rho}{\partial \psi_\mu}\psi^{\nu} - \frac{\partial^R Q^\rho}{\partial \psi_\nu}\psi^{\mu} + \frac{1}{2}\frac{\partial^R Q^\rho}{\partial \psi_\lambda}\gamma^{\mu\nu}\psi^\lambda = 0, \tag{193}$$

which shows that (see (175) and (187))

$$Q^\rho = Q^\rho(X, Y, Z). \tag{194}$$

Since Q^μ like in (194) must provide $V^{\mu\lambda}$ via taking its right derivative with respect to ψ_λ (see (190)), it results that

$$Q^\mu = Q(X, Y, Z)\gamma^\mu, \tag{195}$$

with $Q(X, Y, Z)$ being an arbitrary polynomial. Formulas (190) and (195) together with some appropriate Fierz identities further yield

$$V^{\mu\nu} = \bar{\psi}_\rho P^{\rho\mu\nu}(X, Y, Z), \tag{196}$$

where

$$P^{\rho\mu\nu}(X, Y, Z) = \left(P^{\rho\mu\nu}\right)_\alpha (X, Y, Z)\gamma^\alpha + \left(P^{\rho\mu\nu}\right)_{\alpha\beta\gamma}(X, Y, Z)\gamma^{\alpha\beta\gamma}. \tag{197}$$

The dependence on X, Y, and Z of the functions $\left(P^{\rho\mu\nu}\right)_\alpha$ and $\left(P^{\rho\mu\nu}\right)_{\alpha\beta\gamma}$ enables us to conclude that the most general form of these coefficients reads as

$$\left(P^{\rho\mu\nu}\right)_\alpha (X, Y, Z) = d_1\delta^\rho_\alpha\sigma^{\mu\nu} + d_2\delta^\mu_\alpha\sigma^{\rho\nu} + d_3\delta^\nu_\alpha\sigma^{\rho\mu}, \tag{198}$$

$$\left(P^{\rho\mu\nu}\right)_{\alpha\beta\gamma}(X, Y, Z) = d_4\delta^\rho_{[\alpha}\delta^\mu_\beta\sigma^\nu_{\gamma]}, \tag{199}$$

where $(d_i)_{i=1,2,3,4}$ are arbitrary polynomials in X, Y, and Z. We remark that (199) gives in (133), and thus in $S_1^{(\mathrm{int})}$, a contribution (up to a trivial, s-exact term) that is already contained in (187) since

$$\begin{aligned}\bar{\psi}_\rho\left(P^{\rho\mu\nu}\right)_{\alpha\beta\gamma}\gamma^{\alpha\beta\gamma}\partial_\mu\psi_\nu &= 6d_4\bar{\psi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho \\ &= s\left(-6\mathrm{i}d_4\bar{\psi}_\mu\bar{\psi}^{*\mu}\right) - 6\mathrm{i}d_4\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu,\end{aligned} \tag{200}$$

so we can take, without loss of generality,

$$d_4 = 0 \tag{201}$$

in (199). Taking into account the last result and inserting (198) in (197) and then in (196), we infer that

$$\begin{aligned}V^{\mu\nu}\partial_\mu\psi_\nu &= d_1\bar{\psi}_\rho\gamma^\rho\partial_\mu\psi^\mu + d_2\bar{\psi}^\nu\gamma^\mu\partial_\mu\psi_\nu + d_3\bar{\psi}^\nu\gamma^\mu\partial_\nu\psi_\mu \\ &= d_1\bar{\psi}_\rho\gamma^\rho\partial_\mu\psi^\mu + \frac{1}{2}(d_2+d_3)\bar{\psi}^\nu\gamma^\mu\partial_{(\mu}\psi_{\nu)} \\ &\quad + \frac{1}{2}(d_3-d_2)\bar{\psi}^\mu\gamma^\nu\partial_{[\mu}\psi_{\nu]} \\ &= d_1\bar{\psi}_\rho\gamma^\rho\partial_\mu\psi^\mu + \frac{1}{2}(d_2+d_3)\bar{\psi}^\nu\gamma^\mu\partial_{(\mu}\psi_{\nu)} \\ &\quad + s\left(-\frac{\mathrm{i}}{4}(d_3-d_2)\left(\bar{\psi}_\mu\gamma^{\mu\nu}\bar{\psi}^*_\nu - \bar{\psi}_\mu\bar{\psi}^{*\mu}\right)\right) \\ &\quad - \frac{\mathrm{i}m}{2}(d_3-d_2)\left(\bar{\psi}_\mu\psi^\mu + \bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu\right).\end{aligned} \tag{202}$$

Thus, up to an irrelevant, s-exact term, $V^{\mu\nu}\partial_\mu\psi_\nu$ contains, besides the first two pieces, the last component, which is a contribution already considered in (187). We can thus forget about it by setting

$$d_3 - d_2 = 0. \tag{203}$$

At this stage, from (201) and (203) replaced in (198)–(199) and the resulting relations further substituted in (197), with the help of the representation (196) we determine the relevant part of $V^{\mu\nu}$ under the form

$$\begin{aligned}V^{\mu\nu} &= d_1(X, Y, Z)\bar{\psi}_\alpha\gamma^\alpha\sigma^{\mu\nu} \\ &\quad + d_2(X, Y, Z)\left(\bar{\psi}^\nu\gamma^\mu + \bar{\psi}^\mu\gamma^\nu\right).\end{aligned} \tag{204}$$

Consequently, we find that $V^{\mu\nu}\partial_\mu\psi_\nu$ no longer contains the unwanted (trivial or redundant) contributions, being precisely given by

$$\begin{aligned}V^{\mu\nu}\partial_\mu\psi_\nu &= d_1(X, Y, Z)\bar{\psi}_\rho\gamma^\rho\partial_\mu\psi^\mu \\ &\quad + d_2(X, Y, Z)\bar{\psi}^\nu\gamma^\mu\partial_{(\mu}\psi_{\nu)}.\end{aligned} \tag{205}$$

Based on the relations (204) and (205), we deduce that the antisymmetric part of $\Pi_1^{\mu\nu}$ must vanish

$$\Pi_1^{\mu\nu} - \Pi_1^{\nu\mu} = 0. \tag{206}$$

As a consequence of this step of the deformation procedure, on the one hand the results (164), (187), and (205) completely determine the component (133), and hence the cross-coupling part of the first-order deformation (136) like

$$\begin{aligned}S_1^{(\mathrm{int})} = \int \mathrm{d}^4x \Big(&\psi^{*\mu}\left(\partial^\nu\psi_\mu\right)\eta_\nu + \frac{1}{2}\psi^{*\mu}\psi^\nu\partial_{[\mu}\eta_{\nu]} \\ &+ \frac{1}{8}\psi^{*\rho}\gamma^{\mu\nu}\psi_\rho\partial_{[\mu}\eta_{\nu]} \\ &+ \frac{1}{2}\left(\sigma^{\rho\lambda}\mathcal{L}_0^{(\mathrm{RS})} - \frac{\mathrm{i}}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\partial^\lambda\psi_\nu\right)h_{\rho\lambda} + \frac{\mathrm{i}}{4} \\ &\times\left(\frac{1}{2}\bar{\psi}^\mu\gamma^\rho\psi^\nu + \sigma^{\mu\rho}\bar{\psi}^\nu\gamma^\sigma\psi_\sigma + \bar{\psi}_\sigma\gamma^{\sigma\rho\mu}\psi^\nu\right)\partial_{[\mu}h_{\nu]\rho} \\ &+ V + d_1\bar{\psi}_\rho\gamma^\rho\partial_\mu\psi^\mu + d_2\bar{\psi}^\nu\gamma^\mu\partial_{(\mu}\psi_{\nu)}\Big).\end{aligned} \tag{207}$$

On the other hand, (168), (174), (187), (204), and (206) offer us the concrete form of $b_0^{(\mathrm{int})}$ as the solution to (149) like

$$
\begin{aligned}
b_0^{(\mathrm{int})} = & \frac{1}{8}\mathcal{L}_0^{(\mathrm{RS})}\left(h^2 - 2h_{\mu\nu}h^{\mu\nu}\right) \\
& - \frac{\mathrm{i}}{16}\bar{\psi}^{\mu}\gamma^{\lambda}\psi^{\nu}\left(\left(h^{\lambda}_{\rho} - h\delta^{\lambda}_{\rho}\right)\partial_{[\mu}h_{\nu]\lambda} \right. \\
& \left. + h^{\sigma}_{\nu}\left(2\partial_{[\mu}h_{\sigma]\lambda} + \partial_{\lambda}h_{\mu\sigma}\right)\right) \\
& - \frac{\mathrm{i}}{8}\bar{\psi}^{\mu}\gamma^{\sigma}\psi_{\sigma}\left(h\left(\partial_{\mu}h - \partial^{\nu}h_{\mu\nu}\right) + h^{\rho}_{\mu}\left(\partial^{\lambda}h_{\rho\lambda} - \partial_{\rho}h\right) \right. \\
& \left. - 2h^{\alpha\beta}\partial_{\mu}h_{\alpha\beta} + \frac{3}{2}h^{\rho\lambda}\partial_{\rho}h_{\mu\lambda} + \frac{1}{2}h_{\mu\nu}\partial_{\rho}h^{\rho\nu}\right) \\
& - \frac{\mathrm{i}}{8}\bar{\psi}_{\mu}\gamma^{\mu\nu\beta}\left(\partial^{\alpha}\psi_{\nu}\right)\left(hh_{\alpha\beta} - \frac{3}{2}h_{\alpha\sigma}h^{\sigma}_{\beta}\right) \\
& - \frac{\mathrm{i}}{16}\bar{\psi}_{\beta}\gamma^{\beta\mu\alpha}\psi^{\nu}\left(\left(h\delta^{\rho}_{\nu} - \frac{1}{2}h^{\rho}_{\nu}\right)\partial_{[\mu}h_{\alpha]\rho} \right. \\
& \left. + h^{\rho}_{\mu}\left(3\partial_{\alpha}h_{\nu\rho} - 2\partial_{\rho}h_{\alpha\nu}\right)\right) \\
& + \frac{h}{2}V + \frac{d_1}{2}\bar{\psi}_{\rho}\gamma^{\rho}\left(h\partial_{\mu}\psi^{\mu} - \left(\partial_{\mu}\psi_{\nu}\right)h^{\mu\nu} \right. \\
& \left. - \sigma^{\mu\nu}\psi^{\sigma}\partial_{[\nu}h_{\sigma]\mu} - \frac{1}{4}\sigma^{\mu\nu}\gamma^{\alpha\beta}\psi_{\nu}\partial_{[\alpha}h_{\beta]\mu}\right) \\
& + \frac{d_2}{2}\bar{\psi}_{\rho}\left(h\gamma^{\mu}\partial_{\mu}\psi^{\rho} - h^{\mu\nu}\gamma_{\mu}\partial_{\nu}\psi^{\rho} \right. \\
& \left. - \gamma_{\mu}\psi_{\lambda}\partial^{[\rho}h^{\lambda]\mu} - \frac{1}{4}\gamma^{\mu}\gamma^{\alpha\beta}\psi^{\rho}\partial_{[\alpha}h_{\beta]\mu}\right) \\
& + \frac{d_2}{2}\left(h\bar{\psi}^{\rho}\gamma^{\mu}\partial_{\rho}\psi_{\mu} - h^{\mu\nu}\bar{\psi}_{\mu}\gamma^{\rho}\partial_{\nu}\psi_{\rho} \right. \\
& \left. - \bar{\psi}^{\mu}\gamma^{\nu}\psi^{\rho}\partial_{[\nu}h_{\rho]\mu} - \frac{1}{4}\bar{\psi}^{\mu}\gamma^{\nu}\gamma^{\alpha\beta}\psi_{\nu}\partial_{[\alpha}h_{\beta]\mu}\right) . \qquad (208)
\end{aligned}
$$

Now, the components $\left(b_I^{(\mathrm{int})}\right)$ $I = 0, 1, 2$ expressed by (166), (167), and (208) yield the cross-coupling part of the second-order deformation $S_2^{(\mathrm{int})} = \int \mathrm{d}^4x \left(b_0^{(\mathrm{int})} + b_1^{(\mathrm{int})} + b_2^{(\mathrm{int})}\right)$ as

$$
\begin{aligned}
S_2^{(\mathrm{int})} = & \int \mathrm{d}^4x \left(\frac{1}{8}\left(\psi^{*[\mu}\psi^{\sigma]} + \frac{1}{2}\psi^{*\rho}\gamma^{\mu\sigma}\psi_{\rho}\right)h^{\lambda}_{\mu}\partial_{[\sigma}\eta_{\lambda]} \right. \\
& - \frac{1}{2}\psi^{*\sigma}\left(\partial^{\mu}\psi_{\sigma}\right)\eta^{\nu}h_{\mu\nu} \\
& - \frac{1}{4}\left(\psi^{*\mu}\psi^{\nu} + \frac{1}{4}\psi^{*\sigma}\gamma^{\mu\nu}\psi_{\sigma}\right)\eta^{\rho}\partial_{[\mu}h_{\nu]\rho} \\
& + \frac{1}{8}\mathcal{L}_0^{(\mathrm{RS})}\left(h^2 - 2h_{\mu\nu}h^{\mu\nu}\right) \\
& - \frac{\mathrm{i}}{16}\bar{\psi}^{\mu}\gamma^{\lambda}\psi^{\nu}\left(\left(h^{\lambda}_{\rho} - h\delta^{\lambda}_{\rho}\right)\partial_{[\mu}h_{\nu]\lambda} \right. \\
& \left. + h^{\sigma}_{\nu}\left(2\partial_{[\mu}h_{\sigma]\lambda} + \partial_{\lambda}h_{\mu\sigma}\right)\right) \\
& - \frac{\mathrm{i}}{8}\bar{\psi}^{\mu}\gamma^{\sigma}\psi_{\sigma}\left(h\left(\partial_{\mu}h - \partial^{\nu}h_{\mu\nu}\right) + h^{\rho}_{\mu}\left(\partial^{\lambda}h_{\rho\lambda} - \partial_{\rho}h\right) \right. \\
& \left. - 2h^{\alpha\beta}\partial_{\mu}h_{\alpha\beta} + \frac{3}{2}h^{\rho\lambda}\partial_{\rho}h_{\mu\lambda} + \frac{1}{2}h_{\mu\nu}\partial_{\rho}h^{\rho\nu}\right) \\
& - \frac{\mathrm{i}}{8}\bar{\psi}_{\mu}\gamma^{\mu\nu\beta}\left(\partial^{\alpha}\psi_{\nu}\right)\left(hh_{\alpha\beta} - \frac{3}{2}h_{\alpha\sigma}h^{\sigma}_{\beta}\right) \\
& - \frac{\mathrm{i}}{16}\bar{\psi}_{\beta}\gamma^{\beta\mu\alpha}\psi^{\nu}\left(\left(h\delta^{\rho}_{\nu} - \frac{1}{2}h^{\rho}_{\nu}\right)\partial_{[\mu}h_{\alpha]\rho} \right. \\
& \left. + h^{\rho}_{\mu}\left(3\partial_{\alpha}h_{\nu\rho} - 2\partial_{\rho}h_{\alpha\nu}\right)\right) \\
& + \frac{h}{2}V + \frac{d_1}{2}\bar{\psi}_{\rho}\gamma^{\rho}\left(h\partial_{\mu}\psi^{\mu} - \left(\partial_{\mu}\psi_{\nu}\right)h^{\mu\nu} \right. \\
& \left. - \sigma^{\mu\nu}\psi^{\sigma}\partial_{[\nu}h_{\sigma]\mu} - \frac{1}{4}\sigma^{\mu\nu}\gamma^{\alpha\beta}\psi_{\nu}\partial_{[\alpha}h_{\beta]\mu}\right) \\
& + \frac{d_2}{2}\bar{\psi}_{\rho}\left(h\gamma^{\mu}\partial_{\mu}\psi^{\rho} - h^{\mu\nu}\gamma_{\mu}\partial_{\nu}\psi^{\rho} \right. \\
& \left. - \gamma_{\mu}\psi_{\lambda}\partial^{[\rho}h^{\lambda]\mu} - \frac{1}{4}\gamma^{\mu}\gamma^{\alpha\beta}\psi^{\rho}\partial_{[\alpha}h_{\beta]\mu}\right) \\
& + \frac{d_2}{2}\left(h\bar{\psi}^{\rho}\gamma^{\mu}\partial_{\rho}\psi_{\mu} \right. \\
& - h^{\mu\nu}\bar{\psi}_{\mu}\gamma^{\rho}\partial_{\nu}\psi_{\rho} - \bar{\psi}^{\mu}\gamma^{\nu}\psi^{\rho}\partial_{[\nu}h_{\rho]\mu} \\
& \left.\left. - \frac{1}{4}\bar{\psi}^{\mu}\gamma^{\nu}\gamma^{\alpha\beta}\psi_{\nu}\partial_{[\alpha}h_{\beta]\mu}\right)\right) . \qquad (209)
\end{aligned}
$$

This ends the second step of the deformation procedure for the Pauli–Fierz field and the massive Rarita–Schwinger field.

5 Lagrangian formulation of the interacting theory

The main aim of this section is to give an appropriate interpretation of the Lagrangian formulation of the interacting theory obtained in the previous section from the deformation of the solution to the master equation. In view of this, we initially prove that the linearized versions of first- and second-order formulations of spin-two field theory possess isomorphic local BRST cohomologies. We start from the first-order formulation of spin-two field theory

$$
\begin{aligned}
S\left[e_a{}^{\mu}, \omega_{\mu ab}\right] = & -\frac{1}{\lambda}\int \mathrm{d}^4x \left(\omega_{\nu}{}^{ab}\partial_{\mu}\left(ee_a{}^{\mu}e_b{}^{\nu}\right) \right. \\
& - \omega_{\mu}{}^{ab}\partial_{\nu}\left(ee_a{}^{\mu}e_b{}^{\nu}\right) \\
& \left. + \frac{1}{2}ee_a{}^{\mu}e_b{}^{\nu}\left(\omega_{\mu}{}^{ac}\omega_{\nu}{}^{b}{}_{c} - \omega_{\nu}{}^{ac}\omega_{\mu}{}^{b}{}_{c}\right)\right) , \qquad (210)
\end{aligned}
$$

where $e_a{}^{\mu}$ is the vierbein field and $\omega_{\mu ab}$ are the components of the spin connection, while e is the inverse of the vierbein determinant

$$e = \left(\det\left(e_a{}^{\mu}\right)\right)^{-1} . \qquad (211)$$

In order to linearize action (210), we develop the vierbein as

$$e_a{}^{\mu} = \delta_a{}^{\mu} - \frac{\lambda}{2}f_a{}^{\mu}, \quad e = 1 + \frac{\lambda}{2}f , \qquad (212)$$

where f is the trace of $f_a{}^\mu$. Consequently, we find that the linearized form of (210) reads as (we come back to the notations μ, ν, etc. for flat indices)

$$S_0'\left[f_{\mu\nu}\,,\omega_{\mu\alpha\beta}\right]=\int \mathrm{d}^4x\bigg(\omega_\alpha{}^{\alpha\mu}\left(\partial_\mu f-\partial^\nu f_{\mu\nu}\right)+\frac{1}{2}\omega^{\mu\alpha\beta}\partial_{[\alpha}f_{\beta]\mu}-\frac{1}{2}\left(\omega_\alpha{}^{\alpha\beta}\omega^\lambda{}_{\lambda\beta}-\omega^{\mu\alpha\beta}\omega_{\alpha\mu\beta}\right)\bigg)\,. \tag{213}$$

We mention that the field $f_{\mu\nu}$ contains a symmetric, as well as an antisymmetric part. The above linearized action is invariant under the gauge transformations

$$\delta_\epsilon f_{\mu\nu}=\partial_\mu\epsilon_\nu-\epsilon_{\mu\nu}\,,\quad \delta_\epsilon\omega_{\mu\alpha\beta}=-\partial_\mu\epsilon_{\alpha\beta}\,, \tag{214}$$

where the latter gauge parameters are antisymmetric, $\epsilon_{\alpha\beta}=-\epsilon_{\beta\alpha}$. Eliminating the spin connection components on their equations of motion (auxiliary fields) from (213)

$$\omega_{\mu\alpha\beta}\left(f\right)=\frac{1}{2}\left(\partial_{[\mu}f_{\alpha]\beta}-\partial_{[\mu}f_{\beta]\alpha}-\partial_{[\alpha}f_{\beta]\mu}\right)\,, \tag{215}$$

we obtain the second-order action

$$\begin{aligned}S_0'\left[f_{\mu\nu}\,,\omega_{\mu\alpha\beta}\left(f\right)\right]&=S_0''\left[f_{\mu\nu}\right]\\&=-\int \mathrm{d}^4x\bigg(\frac{1}{8}\left(\partial^{[\mu}f^{\nu]\alpha}\right)\left(\partial_{[\mu}f_{\nu]\alpha}\right)+\frac{1}{4}\left(\partial^{[\mu}f^{\nu]\alpha}\right)\left(\partial_{[\mu}f_{\alpha]\nu}\right)-\frac{1}{2}\left(\partial_\mu f-\partial^\nu f_{\mu\nu}\right)\left(\partial^\mu f-\partial_\alpha f^{\mu\alpha}\right)\bigg)\,,\end{aligned} \tag{216}$$

subject to the gauge invariances

$$\delta_\epsilon f_{\mu\nu}=\partial_{(\mu}\epsilon_{\nu)}-\epsilon_{\mu\nu}\,. \tag{217}$$

If we decompose $f_{\mu\nu}$ in its symmetric and antisymmetric parts

$$f_{\mu\nu}=h_{\mu\nu}+B_{\mu\nu}\,,\quad h_{\mu\nu}=h_{\nu\mu}\,,\quad B_{\mu\nu}=-B_{\nu\mu}\,, \tag{218}$$

the action (216) becomes

$$\begin{aligned}S_0''\left[f_{\mu\nu}\right]&=S_0''\left[h_{\mu\nu}\,,B_{\mu\nu}\right]\\&=\int \mathrm{d}^4x\bigg(-\frac{1}{2}\left(\partial_\mu h_{\nu\rho}\right)\left(\partial^\mu h^{\nu\rho}\right)+\left(\partial_\mu h^{\mu\rho}\right)\left(\partial^\nu h_{\nu\rho}\right)-\left(\partial_\mu h\right)\left(\partial_\nu h^{\nu\mu}\right)+\frac{1}{2}\left(\partial_\mu h\right)\left(\partial^\mu h\right)\bigg)\,,\end{aligned} \tag{219}$$

while the accompanying gauge transformations are given by

$$\delta_\epsilon h_{\mu\nu}=\partial_{(\mu}\epsilon_{\nu)}\,,\quad \delta_\epsilon B_{\mu\nu}=-\epsilon_{\mu\nu}\,. \tag{220}$$

It is easy to see that the right-hand side of (219) is nothing but the Pauli–Fierz action

$$S_0''\left[h_{\mu\nu}\,,B_{\mu\nu}\right]=S_0^{\mathrm{PF}}\left[h_{\mu\nu}\right]\,. \tag{221}$$

As we have previously mentioned, we pass from (213)–(214) to (219)–(220) via the elimination of the auxiliary fields $\omega_{\mu\alpha\beta}$, such that the general theorems from Section 15 of the first reference in [23] ensure the isomorphism

$$H\left(s'|d\right)\simeq H\left(s''|d\right)\,, \tag{222}$$

with s' and s'' the BRST differentials corresponding to (213)–(214) and to (219)–(220), respectively. On the other hand, we observe that the field $B_{\mu\nu}$ does not appear in (219) and is subject to a shift gauge symmetry. Thus, in any cohomological class from $H\left(s''|d\right)$ one can take a representative that is independent of $B_{\mu\nu}$, the shift ghosts as well as of their antifields. This is because these variables form contractible pairs that drop out from $H\left(s''|d\right)$ (see the general results in Section 14 of the first reference in [23]). As a consequence, we have that

$$H\left(s''|d\right)\simeq H\left(s|d\right)\,, \tag{223}$$

where s is the Pauli–Fierz BRST differential. Combining (222) and (223), we arrive at

$$H\left(s'|d\right)\simeq H\left(s''|d\right)\simeq H\left(s|d\right)\,. \tag{224}$$

Because the local BRST cohomology (in ghost number equal to zero and one) controls the deformation procedure, it results that the last isomorphisms allow one to pass in a consistent manner from the Pauli–Fierz version to the first- and second-order ones (in vierbein formulation) during the deformation procedure.

It is easy to see that one can go from (219)–(220) to the Pauli–Fierz version through the partial gauge-fixing $B_{\mu\nu}=0$. This gauge-fixing is a consequence of the more general gauge-fixing condition [27]

$$\sigma_{\mu[a}e_{b]}{}^\mu=0\,. \tag{225}$$

In the context of the larger partial gauge-fixing (225), simple computation leads to the vierbein fields $e_a{}^\mu$, their inverse $e^a{}_\mu$, the inverse of their determinant e, and the components of the spin connection $\omega_{\mu ab}$ up to the second order in the coupling constant in terms of the Pauli–Fierz field as

$$\begin{aligned}e_a{}^\mu&=\overset{(0)}{e}{}_a{}^\mu+\lambda\overset{(1)}{e}{}_a{}^\mu+\lambda^2\overset{(2)}{e}{}_a{}^\mu+\ldots\\&=\delta_a{}^\mu-\frac{\lambda}{2}h_a{}^\mu+\frac{3\lambda^2}{8}h_a{}^\rho h_\rho{}^\mu+\ldots\,,\end{aligned} \tag{226}$$

$$\begin{aligned}e^a{}_\mu&=\overset{(0)}{e}{}^a{}_\mu+\lambda\overset{(1)}{e}{}^a{}_\mu+\lambda^2\overset{(2)}{e}{}^a{}_\mu+\ldots\\&=\delta^a{}_\mu+\frac{\lambda}{2}h^a{}_\mu-\frac{\lambda^2}{8}h^a{}_\rho h^\rho{}_\mu+\ldots\,,\end{aligned} \tag{227}$$

$$\begin{aligned}e&=\overset{(0)}{e}+\lambda\overset{(1)}{e}+\lambda^2\overset{(2)}{e}+\ldots\\&=1+\frac{\lambda}{2}h+\frac{\lambda^2}{8}\left(h^2-2h_{\mu\nu}h^{\mu\nu}\right)+\ldots\,,\end{aligned} \tag{228}$$

$$\omega_{\mu ab}=\lambda\overset{(1)}{\omega}{}_{\mu ab}+\lambda^2\overset{(2)}{\omega}{}_{\mu ab}+\ldots\,, \tag{229}$$

where

$$\overset{(1)}{\omega}{}_{\mu ab}=-\partial_{[a}h_{b]\mu}\,, \tag{230}$$

$$\overset{(2)}{\omega}_{\mu ab} = -\frac{1}{4}\left(2h_{c[a}\left(\partial_{b]}h^{c}{}_{\mu}\right) - 2h_{[a}{}^{\nu}\partial_{\nu}h_{b]\mu} - \left(\partial_{\mu}h_{[a}{}^{\nu}\right)h_{b]\nu}\right). \tag{231}$$

Based on the isomorphisms (224), we can further pass to the analysis of the deformed theory obtained in the previous sections.

The component of antighost number equal to zero in $S_1^{(\mathrm{int})}$ is precisely the interacting Lagrangian at order one in the coupling constant $\mathcal{L}_1^{(\mathrm{int})} = a_0^{(\mathrm{int})} + a_0^{(\mathrm{RS})}$

$$\begin{aligned}
\mathcal{L}_1^{(\mathrm{int})} = & \left[\frac{1}{4}\bar{\psi}_\mu\left(-\mathrm{i}\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho + m\gamma^{\mu\nu}\psi_\nu\right)h\right] \\
& + \left[\frac{\mathrm{i}}{4}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\left(\partial^\lambda\psi_\rho\right)h_{\nu\lambda}\right] \\
& + \left[\frac{\mathrm{i}}{4}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\left(\partial_\nu\psi^\lambda\right)h_{\rho\lambda}\right] \\
& + \left[\frac{\mathrm{i}}{8}\left(\bar{\psi}^\mu\gamma^\lambda\psi^\nu - 2\sigma^{\nu\lambda}\bar{\psi}^\mu\gamma^\rho\psi_\rho\right)\partial_{[\mu}h_{\nu]\lambda}\right] \\
& + \left[-\frac{\mathrm{i}}{8}\left(2\bar{\psi}_\mu\gamma^{\mu\nu\rho}\left(\partial_\nu\psi^\lambda\right)h_{\rho\lambda} + \bar{\psi}_\rho\gamma^{\rho\mu\nu}\psi^\lambda\partial_{[\mu}h_{\nu]\lambda}\right)\right] \\
& + [V] + \left[d_1\bar{\psi}_\rho\gamma^\rho\partial_\mu\psi^\mu\right] + \left[d_2\bar{\psi}^{(\mu}\gamma^{\nu)}\partial_\mu\psi_\nu\right] \\
\equiv & \overset{(1)}{e}\mathcal{L}_0^{(\mathrm{RS})} + \overset{(0)}{e}\overset{(1)}{e}{}_b{}^\mu\overset{(0)}{e}{}_c{}^\nu\left(-\frac{\mathrm{i}}{2}\bar{\psi}_a\gamma^{abc}\overset{(0)}{D}_\mu\overset{(0)}{\psi}_\nu\right) \\
& + \overset{(0)}{e}{}_b{}^\mu\overset{(1)}{e}{}_c{}^\nu\left(-\frac{\mathrm{i}}{2}\bar{\psi}_a\gamma^{abc}\overset{(0)}{D}_\mu\overset{(0)}{\psi}_\nu\right) \\
& + \overset{(0)}{e}\overset{(0)}{e}{}_b{}^\mu\overset{(0)}{e}{}_c{}^\nu\left(-\frac{\mathrm{i}}{2}\bar{\psi}_a\gamma^{abc}\overset{(1)}{D}_\mu\overset{(0)}{\psi}_\nu\right) \\
& + \overset{(0)}{e}\overset{(0)}{e}{}_b{}^\mu\overset{(0)}{e}{}_c{}^\nu\left(-\frac{\mathrm{i}}{2}\bar{\psi}_a\gamma^{abc}\overset{(0)}{D}_\mu\overset{(1)}{\psi}_\nu\right) \\
& + \overset{(0)}{e}V + d_1\bar{\psi}_a\gamma^a\overset{(0)}{D}_\mu\left(\overset{(0)}{e}\overset{(0)}{\psi}{}^\mu\right) \\
& + d_2\overset{(0)}{e}\bar{\psi}^{(a}\gamma^{b)}\overset{(0)}{e}{}_b{}^\mu\overset{(0)}{e}{}_c{}^\nu\overset{(0)}{D}_\mu\left(\overset{(0)}{e}\overset{(0)}{\psi}_\nu\right),
\end{aligned} \tag{232}$$

where

$$\overset{(0)}{D}_\mu = \partial_\mu\,, \tag{233}$$

and

$$\overset{(1)}{D}_\mu = \frac{1}{8}\overset{(1)}{\omega}_{\mu ab}\gamma^{ab}\,, \tag{234}$$

with $\overset{(1)}{\omega}_{\mu ab}$ given in (230). Along the same lines, the piece of antighost number equal to zero from the second-order deformation offers us the interacting Lagrangian at order two in the coupling constant $\mathcal{L}_2^{(\mathrm{int})} = b_0^{(\mathrm{int})}$

$$\begin{aligned}
\mathcal{L}_2^{(\mathrm{int})} = & b_0^{(\mathrm{int})} \\
= & \left[\frac{1}{16}\bar{\psi}_\mu\left(-\mathrm{i}\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho + m\gamma^{\mu\nu}\psi_\nu\right)\left(h^2 - 2h_{\alpha\beta}h^{\alpha\beta}\right)\right] \\
& + \left[\frac{\mathrm{i}}{8}\bar{\psi}_\mu\left(\gamma^{\mu\alpha\nu}\left(\partial^\beta\psi_\nu\right)h_{\alpha\beta} + \gamma^{\mu\nu\rho}\left(\partial_\nu\psi^\lambda\right)h_{\rho\lambda}\right)h\right] \\
& + \left[\frac{\mathrm{i}h}{16}\left(-\bar{\psi}_\mu\gamma^{\mu\nu\rho}\left(2\left(\partial_\nu\psi^\lambda\right)h_{\rho\lambda} + \psi^\lambda\partial_{[\nu}h_{\rho]\lambda}\right)\right.\right. \\
& \left.\left. + \left(\bar{\psi}^\alpha\gamma^\rho\psi^\beta - 2\sigma^{\beta\rho}\bar{\psi}^\alpha\gamma^\mu\psi_\mu\right)\partial_{[\alpha}h_{\beta]\rho}\right)\right] \\
& + \left[-\frac{\mathrm{i}}{8}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\left(\partial_\alpha\psi_\beta\right)h_\nu^\alpha h_\rho^\beta\right] \\
& + \left[\frac{\mathrm{i}}{8}\left(\bar{\psi}_\alpha\gamma^{\alpha\beta\gamma}\left(h_\beta^\mu\partial_\mu\left(h_\gamma^\sigma\psi_\sigma\right) + h_\gamma^\mu\partial_\beta\left(h_\mu^\sigma\psi_\sigma\right)\right)\right.\right. \\
& \left.\left. -\frac{1}{2}\left(\bar{\psi}^\mu\gamma_\rho\psi^\nu h^{\rho\sigma} - 2\bar{\psi}^\mu\gamma^\rho\psi_\rho h^{\nu\sigma}\right)\partial_{[\mu}h_{\nu]\sigma}\right)\right] \\
& + \left[\frac{\mathrm{i}}{8}\left(\bar{\psi}_\alpha\gamma^{\alpha\beta\gamma}\partial_\beta\left(h_\gamma^\mu h_\mu^\sigma\psi_\sigma\right)\right.\right. \\
& -\frac{1}{2}\bar{\psi}^\mu\left(\gamma^\rho\psi_\rho\left(3h_{\mu\lambda}\partial_\sigma h^{\lambda\sigma}\right.\right. \\
& \left. + h^{\lambda\sigma}\partial_\lambda h_{\mu\sigma} - 2h_{\mu\sigma}\partial^\sigma h - 2h^{\alpha\beta}\partial_\mu h_{\alpha\beta}\right) \\
& \left.\left.\left. -\gamma^\lambda\psi^\nu\left(2h_{\rho\mu}\partial_\nu h_\lambda^\rho - 2h_\mu^\rho\partial_\rho h_{\nu\lambda} - h_{\nu\rho}\partial_\lambda h_\mu^\rho\right)\right)\right)\right] \\
& + \left[\frac{3\mathrm{i}}{16}\bar{\psi}_\mu\gamma^{\mu\nu\beta}\left(\partial^\alpha\psi_\nu\right)h_{\alpha\sigma}h_\beta^\sigma\right] \\
& + \left[-\frac{3\mathrm{i}}{16}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\left(\partial_\nu\psi_\lambda\right)h_{\rho\sigma}h^{\sigma\lambda}\right] \\
& + \left[\frac{h}{2}V\right] + \left[d_1\bar{\psi}_\rho\gamma^\rho\partial_\mu\left(\frac{h}{2}\psi^\mu\right)\right] \\
& + \left[-\frac{d_1}{2}\bar{\psi}_\rho\gamma^\rho\partial_\mu\left(\psi_\nu h^{\mu\nu}\right)\right] \\
& + \left[-\frac{d_1}{8}\bar{\psi}_\rho\gamma^\rho\gamma^{\alpha\beta}\psi^\mu\partial_{[\alpha}h_{\beta]\mu}\right] + \left[\frac{d_2}{2}h\bar{\psi}^{(\mu}\gamma^{\nu)}\partial_\mu\psi_\nu\right] \\
& + \left[-\frac{d_2}{2}h_\alpha^\mu\bar{\psi}^{(\alpha}\gamma^{\nu)}\partial_\mu\psi_\nu\right] \\
& + \left[-d_2\bar{\psi}^{(\mu}\gamma^{\nu)}\left(\frac{1}{2}\psi^\rho\partial_{[\nu}h_{\rho]\mu} + \frac{1}{8}\gamma^{\alpha\beta}\psi_\nu\partial_{[\alpha}h_{\beta]\mu}\right)\right] \\
\equiv & \left[\overset{(2)}{e}\mathcal{L}_0^{(\mathrm{RS})}\right] + \left[\overset{(1)}{e}\left(\overset{(0)}{e}{}_b{}^\mu\overset{(1)}{e}{}_c{}^\nu + \overset{(1)}{e}{}_b{}^\mu\overset{(0)}{e}{}_c{}^\nu\right)\right. \\
& \left. \times\left(-\frac{\mathrm{i}}{2}\bar{\psi}_a\gamma^{abc}\overset{(0)}{D}_\mu\overset{(0)}{\psi}_\nu\right)\right] \\
& + \left[\overset{(1)}{e}\overset{(0)}{e}{}_b{}^\mu\overset{(0)}{e}{}_c{}^\nu\left(-\frac{\mathrm{i}}{2}\bar{\psi}_a\gamma^{abc}\left(\overset{(0)}{D}_\mu\overset{(1)}{\psi}_\nu + \overset{(1)}{D}_\mu\overset{(0)}{\psi}_\nu\right)\right)\right] \\
& + \left[\overset{(0)}{e}\overset{(1)}{e}{}_b{}^\mu\overset{(1)}{e}{}_c{}^\nu\left(-\frac{\mathrm{i}}{2}\bar{\psi}_a\gamma^{abc}\overset{(0)}{D}_\mu\overset{(0)}{\psi}_\nu\right)\right] \\
& + \left[\overset{(0)}{e}\overset{(1)}{e}{}_b{}^\mu\overset{(0)}{e}{}_c{}^\nu\left(-\frac{\mathrm{i}}{2}\bar{\psi}_a\gamma^{abc}\left(\overset{(0)}{D}_\mu\overset{(1)}{\psi}_\nu + \overset{(1)}{D}_\mu\overset{(0)}{\psi}_\nu\right)\right)\right. \\
& \left. + \overset{(0)}{e}\overset{(0)}{e}{}_b{}^\mu\overset{(1)}{e}{}_c{}^\nu\left(-\frac{\mathrm{i}}{2}\bar{\psi}_a\gamma^{abc}\left(\overset{(0)}{D}_\mu\overset{(1)}{\psi}_\nu + \overset{(1)}{D}_\mu\overset{(0)}{\psi}_\nu\right)\right)\right] \\
& + \left[\overset{(0)}{e}\overset{(0)}{e}{}_b{}^\mu\overset{(0)}{e}{}_c{}^\nu\left(-\frac{\mathrm{i}}{2}\bar{\psi}_a\gamma^{abc}\left(\overset{(0)}{D}_\mu\overset{(2)}{\psi}_\nu + \overset{(1)}{D}_\mu\overset{(1)}{\psi}_\nu\right.\right.\right.
\end{aligned}$$

$$
\begin{aligned}
&+\overset{(2)}{D}_\mu\overset{(0)}{\psi}_\nu\Big)\Big)\Big)\Big]\\
&+\left[\overset{(0)}{e}\,\overset{(2)}{e}{}_b{}^\mu\,\overset{(0)}{e}{}_c{}^\nu\left(-\frac{\mathrm{i}}{2}\bar{\psi}_a\gamma^{abc}\overset{(0)}{D}_\mu\overset{(0)}{\psi}_\nu\right)\right]\\
&+\left[\overset{(0)}{e}\,\overset{(0)}{e}{}_b{}^\mu\,\overset{(2)}{e}{}_c{}^\nu\left(-\frac{\mathrm{i}}{2}\bar{\psi}_a\gamma^{abc}\overset{(0)}{D}_\mu\overset{(0)}{\psi}_\nu\right)\right]+\left[\overset{(1)}{e}V\right]\\
&+\left[d_1\bar{\psi}_a\gamma^a\overset{(0)}{D}_\mu\left(\overset{(1)}{e}\,\overset{(0)}{\psi}{}^\mu\right)\right]\\
&+\left[d_1\bar{\psi}_a\gamma^a\overset{(0)}{D}_\mu\left(\overset{(0)}{e}\,\overset{(1)}{\psi}{}^\mu\right)\right]\\
&+\left[d_1\bar{\psi}_a\gamma^a\overset{(1)}{D}_\mu\left(\overset{(0)}{e}\,\overset{(0)}{\psi}{}^\mu\right)\right]\\
&+\left[d_2\overset{(1)}{e}\,\overset{(0)}{e}{}_a{}^\mu\bar{\psi}^{(a}\gamma^{b)}\overset{(0)}{D}_\mu\psi_b\right]\\
&+\left[d_2\overset{(0)}{e}\,\overset{(1)}{e}{}_a{}^\mu\bar{\psi}^{(a}\gamma^{b)}\overset{(0)}{D}_\mu\psi_b\right]\\
&+\left[d_2\overset{(0)}{e}\,\overset{(0)}{e}{}_a{}^\mu\bar{\psi}^{(a}\gamma^{b)}\overset{(1)}{D}_\mu\psi_b\right],
\end{aligned}
\tag{235}
$$

where

$$
\overset{(2)}{D}_\mu=\frac{1}{8}\overset{(2)}{\omega}_{\mu ab}\gamma^{ab}
\tag{236}
$$

and $\overset{(2)}{\omega}_{\mu ab}$ like in (231). With the help of (226) and (228) we deduce that $\mathcal{L}_0^{(\mathrm{RS})}+\lambda\mathcal{L}_1^{(\mathrm{int})}+\lambda^2\mathcal{L}_2^{(\mathrm{int})}+\cdots$ comes from expanding the fully deformed Lagrangian written in terms of either the original flat Rarita–Schwinger spinor ψ_a

$$
\begin{aligned}
\mathcal{L}^{(\mathrm{int})}=&\frac{e}{2}\left(-\mathrm{i}\bar{\psi}_a e_b{}^\nu e_c{}^\rho\gamma^{abc}D_\nu\left(e^d{}_\rho\psi_d\right)+m\bar{\psi}_a\gamma^{ab}\psi_b\right)\\
&+\lambda\Big[eV\left(X,Y,Z\right)+d_1\left(X,Y,Z\right)\bar{\psi}_a\gamma^a D_\mu\left(ee_b{}^\mu\psi^b\right)\\
&+ed_2\left(X,Y,Z\right)e_a{}^\mu\bar{\psi}^{(a}\gamma^{b)}D_\mu\psi_b\Big],
\end{aligned}
\tag{237}
$$

or the curved Rarita–Schwinger spinor ψ_μ

$$
\begin{aligned}
\mathcal{L}^{(\mathrm{int})}=&\frac{e}{2}\left(-\mathrm{i}\bar{\psi}_\mu e_a{}^\mu e_b{}^\nu e_c{}^\rho\gamma^{abc}D_\nu\psi_\rho+m\bar{\psi}_\mu e_a{}^\mu\gamma^{ab}e_b{}^\nu\psi_\nu\right)\\
&+\lambda\left[eV\left(X,Y,Z\right)+d_1\left(X,Y,Z\right)e_a{}^\nu\bar{\psi}_\nu\gamma^a D_\mu\left(e\psi^\mu\right)\right.\\
&\left.+ed_2\left(X,Y,Z\right)\left(\bar{\psi}^\mu\gamma^b+e_a{}^\mu e^b{}_\rho\bar{\psi}^\rho\gamma^a\right)D_\mu\left(e_b{}^\nu\psi_\nu\right)\right].
\end{aligned}
\tag{238}
$$

The notations $D_\mu\psi_a$ and $D_\mu\psi_\rho$ denote the full covariant derivatives of ψ_a and ψ_ρ, respectively

$$
D_\mu\psi_a=\partial_\mu\psi_a+\frac{1}{2}\omega_{\mu ab}\psi^b+\frac{1}{8}\gamma^{bc}\psi_a\omega_{\mu bc},
\tag{239}
$$

$$
D_\mu\psi_\rho=\partial_\mu\psi_\rho+\frac{1}{8}\omega_{\mu ab}\gamma^{ab}\psi_\rho.
\tag{240}
$$

The pieces linear in the antifields ψ_μ^* from the deformed solution to the master equation give us the deformed gauge transformations for the Rarita–Schwinger fields as

$$
\begin{aligned}
\delta_\epsilon\psi_\mu=&\lambda\left(\left(\partial^\alpha\psi_\mu\right)\epsilon_\alpha+\frac{1}{2}\psi^\nu\partial_{[\mu}\epsilon_{\nu]}+\frac{1}{8}\gamma^{\alpha\beta}\psi_\mu\partial_{[\alpha}\epsilon_{\beta]}\right)\\
&+\lambda^2\left(-\frac{1}{2}\left(\partial_\alpha\psi_\mu\right)\epsilon_\beta h^{\alpha\beta}+\frac{1}{16}\gamma^{\rho\lambda}\psi_\mu h^\sigma_\rho\partial_{[\lambda}\epsilon_{\sigma]}\right.\\
&+\frac{1}{8}\psi^\rho\left(h^\lambda_\mu\partial_{[\rho}\epsilon_{\lambda]}-h^\lambda_\rho\partial_{[\mu}\epsilon_{\lambda]}\right)-\frac{1}{4}\psi^\nu\epsilon^\rho\partial_{[\mu}h_{\nu]\rho}\\
&\left.-\frac{1}{16}\gamma^{\alpha\beta}\psi_\mu\epsilon^\rho\partial_{[\alpha}h_{\beta]\rho}\right)\\
=&\lambda\overset{(1)}{\delta}{}_\epsilon\psi_\mu+\lambda^2\overset{(2)}{\delta}{}_\epsilon\psi_\mu+\ldots.
\end{aligned}
\tag{241}
$$

The first two orders of the gauge transformations can be put under the form

$$
\overset{(1)}{\delta}{}_\epsilon\psi_m=\left(\partial_\mu\psi_m\right)\overset{(0)}{\bar{\epsilon}}{}^\mu+\frac{1}{2}\overset{(0)}{\epsilon}{}_{mn}\psi^n+\frac{1}{4}\gamma^{ab}\psi_m\overset{(0)}{\epsilon}{}_{ab},
\tag{242}
$$

$$
\overset{(2)}{\delta}{}_\epsilon\psi_m=\left(\partial_\mu\psi_m\right)\overset{(1)}{\bar{\epsilon}}{}^\mu+\frac{1}{2}\overset{(1)}{\epsilon}{}_{mn}\psi^n+\frac{1}{4}\gamma^{ab}\psi_m\overset{(1)}{\epsilon}{}_{ab},
\tag{243}
$$

where we used the notations

$$
\overset{(0)}{\bar{\epsilon}}{}^\mu=\epsilon^\mu=\epsilon^a\delta_a{}^\mu,\quad\overset{(1)}{\bar{\epsilon}}{}^\mu=-\frac{1}{2}\epsilon^a h_a{}^\mu,
\tag{244}
$$

$$
\overset{(0)}{\epsilon}{}_{ab}=\frac{1}{2}\partial_{[a}\epsilon_{b]},
\tag{245}
$$

$$
\overset{(1)}{\epsilon}{}_{ab}=-\frac{1}{4}\epsilon^c\partial_{[a}h_{b]c}+\frac{1}{8}h^c_{[a}\partial_{b]}\epsilon_c+\frac{1}{8}\left(\partial_c\epsilon_{[a}\right)h^c_{b]}.
\tag{246}
$$

Based on these notations, the gauge transformations of the spinors take the form

$$
\begin{aligned}
\delta_\epsilon\psi_m=\lambda\Bigg(&\left(\partial_\mu\psi_m\right)\left(\overset{(0)}{\bar{\epsilon}}{}^\mu+\lambda\overset{(1)}{\bar{\epsilon}}{}^\mu+\ldots\right)\\
&+\left(\overset{(0)}{\epsilon}{}_{mn}+\lambda\overset{(1)}{\epsilon}{}_{mn}+\ldots\right)\psi^n\\
&+\frac{1}{4}\gamma^{ab}\psi_m\left(\overset{(0)}{\epsilon}{}_{ab}+\lambda\overset{(1)}{\epsilon}{}_{ab}+\ldots\right)\Bigg).
\end{aligned}
\tag{247}
$$

The gauge parameters $\overset{(0)}{\epsilon}{}_{ab}$ and $\overset{(1)}{\epsilon}{}_{ab}$ are precisely the first two terms from the Lorentz parameters expressed in terms of the flat parameters ϵ^a via the partial gauge-fixing (225). Indeed, (225) leads to

$$
\bar{\delta}_\epsilon\sigma_{\mu[a}e_{b]}{}^\mu=0,
\tag{248}
$$

where

$$
\bar{\delta}_\epsilon e_a{}^\mu=\bar{\epsilon}^\rho\partial_\rho e_a{}^\mu-e_a{}^\rho\partial_\rho\bar{\epsilon}^\mu+\epsilon_a{}^b e_b{}^\mu.
\tag{249}
$$

Substituting (226) together with the expansions

$$
\bar{\epsilon}^\mu=\overset{(0)}{\bar{\epsilon}}{}^\mu+\lambda\overset{(1)}{\bar{\epsilon}}{}^\mu+\ldots=\left(\delta_a{}^\mu-\frac{\lambda}{2}h_a{}^\mu+\ldots\right)\epsilon^a
\tag{250}
$$

and

$$\epsilon_{ab} = \overset{(0)}{\epsilon}_{ab} + \lambda \overset{(1)}{\epsilon}_{ab} + \ldots \tag{251}$$

in (248), we arrive precisely at (245)–(246). At this point it is easy to see that the gauge transformations (247) come from the perturbative expansion of the full gauge transformations

$$\delta_\epsilon \psi_m = \lambda \left(\left(\partial_\mu \psi_m\right) \bar{\epsilon}^\mu + \epsilon_{mn} \psi^n + \frac{1}{4} \gamma^{ab} \psi_m \epsilon_{ab} \right). \tag{252}$$

Moreover, based on (252) and (249), it is easy to see that

$$\delta_\epsilon \psi^\mu = \lambda \left(\left(\partial_\sigma \psi^\mu\right) \bar{\epsilon}^\sigma - \psi^\sigma \partial_\sigma \bar{\epsilon}^\mu + \frac{1}{4} \gamma^{ab} \psi^\mu \epsilon_{ab} \right). \tag{253}$$

In conclusion, under the above mentioned hypotheses we have shown that the interactions between a massive Rarita–Schwinger field and a spin-two field are described by the coupled Lagrangian (237) or (238), while the gauge transformations of the Rarita–Schwinger spinors are given by (252) or (253). If we require in addition that the interacting model remains PT-invariant, then the results (237)–(238) remain valid up to the point that the functions V, d_1, and d_2 must depend only on X and Y (and not on Z).

6 Impossibility of cross-interactions between gravitons in the presence of the massive Rarita–Schwinger field

As it has been proved in [16], there are no direct cross-couplings that can be introduced among a finite number of gravitons and also no intermediate cross-couplings between different gravitons in the presence of a scalar field. In this section, under the hypotheses of locality, smoothness of the interactions in the coupling constant, Poincaré invariance, Lorentz covariance, and the preservation of the number of derivatives on each field, we will prove that there are no intermediate cross-couplings between different gravitons intermediated by a massive spin-3/2 field. In order to ensure the stability of the Minkowski vacuum (absence of negative-energy excitations or of negative-norm states) we assume in addition that the metric in internal space is positively defined. It is always possible to bring the internal metric to the form δ_{AB} by a linear redefinition of the Pauli–Fierz fields. This is the convention we will work with in the sequel.

In view of this we start from a finite sum of Pauli–Fierz actions and a massive Rarita–Schwinger action

$$\begin{aligned} S_0^{\mathrm{L}}\left[h_{\mu\nu}^A, \psi_\mu\right] = \int \mathrm{d}^4 x \Big(& -\frac{1}{2}\left(\partial_\mu h_{\nu\rho}^A\right)\left(\partial^\mu h_A^{\nu\rho}\right) \\ & + \left(\partial_\mu h_A^{\mu\rho}\right)\left(\partial^\nu h_{\nu\rho}^A\right) \\ & - \left(\partial_\mu h^A\right)\left(\partial_\nu h_A^{\nu\mu}\right) + \frac{1}{2}\left(\partial_\mu h^A\right)\left(\partial^\mu h_A\right) \Big) \\ & + \int \mathrm{d}^4 x \bar{\psi} \\ & \times \left(-\frac{\mathrm{i}}{2} \bar{\psi}_\mu \gamma^{\mu\nu\rho} \partial_\nu \psi_\rho + \frac{m}{2} \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu \right), \end{aligned} \tag{254}$$

where h_A denotes the trace of the field $h_A^{\mu\nu}$ ($h_A = \sigma_{\mu\nu} h_A^{\mu\nu}$), with A the collection index, running from 1 to n. The gauge transformations of the action (254) read as

$$\delta_\epsilon h_{\mu\nu}^A = \partial_{(\mu} \epsilon_{\nu)}^A, \quad \delta_\epsilon \psi_\mu = 0. \tag{255}$$

The BRST complex comprises the fields/ghosts

$$\phi^{\alpha_0} = \left(h_{\mu\nu}^A, \psi_\mu\right), \quad \eta_\mu^A, \tag{256}$$

and their antifields, respectively,

$$\phi_{\alpha_0}^* = \left(h_A^{*\mu\nu}, \psi^{*\mu}\right), \quad \eta_A^{*\mu}. \tag{257}$$

The BRST differential splits in this situation like in (8), while the actions of δ and γ on the BRST generators are defined by

$$\delta h_A^{*\mu\nu} = 2H_A^{\mu\nu}, \quad \delta \psi^{*\mu} = m \bar{\psi}_\lambda \gamma^{\lambda\mu} - \mathrm{i} \partial_\rho \bar{\psi}_\lambda \gamma^{\rho\lambda\mu}, \tag{258}$$

$$\delta \eta_A^{*\mu} = -2\partial_\nu h_A^{*\mu\nu}, \tag{259}$$

$$\delta \phi^{\alpha_0} = 0, \quad \delta \eta_\mu^A = 0, \tag{260}$$

$$\gamma \phi_{\alpha_0}^* = 0, \quad \gamma \eta_A^{*\mu} = 0, \tag{261}$$

$$\gamma h_{\mu\nu}^A = \partial_{(\mu} \eta_{\nu)}^A, \quad \gamma \psi_\mu = 0, \quad \gamma \eta_\mu^A = 0, \tag{262}$$

where $H_A^{\mu\nu} = K_A^{\mu\nu} - \frac{1}{2}\sigma^{\mu\nu} K_A$ is the linearized Einstein tensor for the field $h_A^{\mu\nu}$. In this case, the solution to the master equation reads as

$$\bar{S} = S_0^{\mathrm{L}}\left[h_{\mu\nu}^A, \psi_\mu\right] + \int \mathrm{d}^4 x \left(h_A^{*\mu\nu} \partial_{(\mu} \eta_{\nu)}^A \right). \tag{263}$$

The first-order deformation of the solution to the master equation may be decomposed in a manner similar to the case of a single graviton

$$\alpha = \alpha^{(\mathrm{PF})} + \alpha^{(\mathrm{int})} + \alpha^{(\mathrm{RS})}. \tag{264}$$

The first-order deformation in the Pauli–Fierz sector, $\alpha^{(\mathrm{PF})}$, is of the form [16]

$$\alpha^{(\mathrm{PF})} = \alpha_2^{(\mathrm{PF})} + \alpha_1^{(\mathrm{PF})} + \alpha_0^{(\mathrm{PF})}, \tag{265}$$

with

$$\alpha_2^{(\mathrm{PF})} = \frac{1}{2} f_{BC}^A \eta_A^{*\mu} \eta^{B\nu} \partial_{[\mu} \eta_{\nu]}^C. \tag{266}$$

In (266) all the coefficients f_{BC}^A are constant. The condition that $\alpha_2^{(\mathrm{PF})}$ indeed produces a consistent $\alpha_1^{(\mathrm{PF})}$ implies that these constants must be symmetric in their lower indices [16][4]

$$f^A_{BC} = f^A_{CB}\,. \qquad (267)$$

With (267) at hand, we find that

$$\alpha_1^{(\mathrm{PF})} = f^A_{BC} h^{*\mu\rho}_A \left(\left(\partial_\rho \eta^{B\nu}\right) h^C_{\mu\nu} - \eta^{B\nu}\partial_{[\mu} h^C_{\nu]\rho}\right). \qquad (268)$$

The requirement that $\alpha_1^{(\mathrm{PF})}$ leads to a consistent $\alpha_0^{(\mathrm{PF})}$ implies that f_{ABC} must be symmetric [16][5]

$$f_{ABC} = \frac{1}{3} f_{(ABC)}\,, \qquad (269)$$

where, by definition, $f_{ABC} = \delta_{AD} f^D_{BC}$. Based on (269), we obtain that the resulting $\alpha_0^{(\mathrm{PF})}$ reads as in [16] (where this component is denoted by a_0 and f_{ABC} by a_{abc}).

If one goes along exactly the same lines as in Sect. 4.2, one obtains that $\alpha^{(\mathrm{int})} = \alpha_1^{(\mathrm{int})} + \alpha_0^{(\mathrm{int})}$, where

$$\alpha_1^{(\mathrm{int})} = k_A \psi^{*\mu}\left(\partial^\nu \psi_\mu\right)\eta^A_\nu + \frac{k_A}{2}\psi^{*\mu}\psi^\nu \partial_{[\mu}\eta^A_{\nu]} + \frac{k_A}{8}\psi^{*\rho}\gamma^{\mu\nu}\psi_\rho \partial_{[\mu}\eta^A_{\nu]}\,, \qquad (270)$$

$$\alpha_0^{(\mathrm{int})} = \frac{k_A}{2}\left(\sigma^{\rho\lambda}\mathcal{L}_0^{(\mathrm{RS})} - \frac{\mathrm{i}}{2}\bar{\psi}_\mu \gamma^{\mu\nu\rho}\partial^\lambda \psi_\nu\right) h^A_{\rho\lambda} + \frac{\mathrm{i}k_A}{4}\left(\frac{1}{2}\bar{\psi}^\mu \gamma^\rho \psi^\nu + \sigma^{\mu\rho}\bar{\psi}^\nu \gamma^\sigma \psi_\sigma + \bar{\psi}_\sigma \gamma^{\sigma\rho\mu}\psi^\nu\right) \times \partial_{[\mu} h^A_{\nu]\rho}\,, \qquad (271)$$

and k_A are some real constants. Meanwhile, we find in a direct manner that

$$\alpha^{(\mathrm{RS})} = a_0^{(\mathrm{RS})}\,, \qquad (272)$$

with $a_0^{(\mathrm{RS})}$ given in (133).

Let us investigate next the consistency of the first-order deformation. If we perform the notations

$$\hat{S}_1^{(\mathrm{PF})} = \int \mathrm{d}^4 x \alpha^{(\mathrm{PF})}\,, \qquad (273)$$

$$\hat{S}_1^{(\mathrm{int})} = \int \mathrm{d}^4 x \left(\alpha^{(\mathrm{int})} + \alpha^{(\mathrm{RS})}\right), \qquad (274)$$

$$\hat{S}_1 = \hat{S}_1^{(\mathrm{PF})} + \hat{S}_1^{(\mathrm{int})}\,, \qquad (275)$$

then the equation $\left(\hat{S}_1, \hat{S}_1\right) + 2s\hat{S}_2 = 0$ (expressing the consistency of the first-order deformation) equivalently splits into two independent equations

$$\left(\hat{S}_1^{(\mathrm{PF})}, \hat{S}_1^{(\mathrm{PF})}\right) + 2s\hat{S}_2^{(\mathrm{PF})} = 0\,, \qquad (276)$$

$$2\left(\hat{S}_1^{(\mathrm{PF})}, \hat{S}_1^{(\mathrm{int})}\right) + \left(\hat{S}_1^{(\mathrm{int})}, \hat{S}_1^{(\mathrm{int})}\right) + 2s\hat{S}_2^{(\mathrm{int})} = 0\,, \qquad (277)$$

[4] The term (266) differs from that corresponding to [16] through a γ-exact term, which does not affect (267).

[5] The piece (268) differs from that corresponding to [16] through a δ-exact term, which does not change (269).

where $\hat{S}_2 = \hat{S}_2^{(\mathrm{PF})} + \hat{S}_2^{(\mathrm{int})}$. (276) requires that the constants f^C_{AB} satisfy the supplementary conditions [16]

$$f^D_{A[B} f^E_{C]D} = 0\,, \qquad (278)$$

thus they are the structure constants of a finite-dimensional, commutative, symmetric, and associative real algebra $\mathcal{A}$. The analysis realized in [16] shows us that such an algebra has a trivial structure (being expressed like a direct sum of some one-dimensional ideals). Thus, we obtain that

$$f^C_{AB} = 0 \quad \text{if} \quad A \neq B\,. \qquad (279)$$

Let us analyze now (277). If we denote by $\hat{\Delta}^{(\mathrm{int})}$ and $\beta^{(\mathrm{int})}$ the non-integrated densities of the functionals $2\left(\hat{S}_1^{(\mathrm{PF})}, \hat{S}_1^{(\mathrm{int})}\right) + \left(\hat{S}_1^{(\mathrm{int})}, \hat{S}_1^{(\mathrm{int})}\right)$ and of $\hat{S}_2^{(\mathrm{int})}$, respectively, then (277) takes the local form

$$\hat{\Delta}^{(\mathrm{int})} = -2s\beta^{(\mathrm{int})} + \partial_\mu k^\mu\,, \qquad (280)$$

with

$$\mathrm{gh}\left(\hat{\Delta}^{(\mathrm{int})}\right) = 1, \quad \mathrm{gh}\left(\beta^{(\mathrm{int})}\right) = 0\,, \quad \mathrm{gh}\left(k^\mu\right) = 1\,. \qquad (281)$$

The computation of $\hat{\Delta}^{(\mathrm{int})}$ reveals in our case the following decomposition along the antighost number

$$\hat{\Delta}^{(\mathrm{int})} = \hat{\Delta}_0^{(\mathrm{int})} + \hat{\Delta}_1^{(\mathrm{int})}\,, \quad \mathrm{agh}\left(\hat{\Delta}_I^{(\mathrm{int})}\right) = I, \quad I = 0, 1, \qquad (282)$$

with

$$\begin{aligned}\hat{\Delta}_1^{(\mathrm{int})} = \gamma \Bigg(&\Bigg(\left(-\frac{1}{4}k_A f^A_{BC}\left(\psi^{*[\mu}\psi^{\sigma]} + \frac{1}{2}\psi^{*\rho}\gamma^{\mu\sigma}\psi_\rho\right) \times \partial_{[\sigma}\eta^B_{\lambda]}\sigma^{\nu\lambda} + \psi^{*\sigma}\left(\partial^\mu \psi_\sigma\right)\eta^{B\nu}\right) h^C_{\mu\nu} \\ &+ \left(k_B k_C - \frac{1}{2}k_A f^A_{BC}\right)\left(\psi^{*\mu}\psi^\nu + \frac{1}{4}\psi^{*\sigma}\gamma^{\mu\nu}\psi_\sigma\right) \times \eta^{B\rho}\partial_{[\mu} h^C_{\nu]\rho}\Bigg) \\ &+ \left(k_A f^A_{BC} - k_B k_C\right)\left(\psi^{*\mu}\left(\partial^\nu \psi_\mu\right)\eta^{B\rho}\partial_{[\nu}\eta^C_{\rho]} + \frac{1}{4}\left(\psi^{*[\mu}\psi^{\nu]} + \frac{1}{2}\psi^{*\sigma}\gamma^{\mu\nu}\psi_\sigma\right)\partial_{[\mu}\eta^B_{\rho]}\partial_{[\nu}\eta^C_{\lambda]}\sigma^{\rho\lambda}\right)\Bigg)\,. \end{aligned} \qquad (283)$$

The concrete form of $\hat{\Delta}_0^{(\mathrm{int})}$ is not important in what follows and, therefore, we will skip it. Due to the expansion (282), we have that $\beta^{(\mathrm{int})}$ and k^μ from (280) split like

$$\beta^{(\mathrm{int})} = \beta_0^{(\mathrm{int})} + \beta_1^{(\mathrm{int})} + \beta_2^{(\mathrm{int})}\,, \quad \mathrm{agh}\left(\beta_I^{(\mathrm{int})}\right) = I, \quad I = 0, 1, 2\,, \qquad (284)$$

$$k^\mu = k_0^\mu + k_1^\mu + k_2^\mu\,, \quad \mathrm{agh}\left(k_I^\mu\right) = I, \quad I = 0, 1, 2\,. \qquad (285)$$

By projecting (280) on the various decreasing values of the antighost number, we obtain the equivalent tower of equations

$$\gamma\beta_2^{(\mathrm{int})} = \partial_\mu \left(\frac{1}{2}k_2^\mu\right), \tag{286}$$

$$\hat{\Delta}_1^{(\mathrm{int})} = -2\left(\delta\beta_2^{(\mathrm{int})} + \gamma\beta_1^{(\mathrm{int})}\right) + \partial_\mu k_1^\mu, \tag{287}$$

$$\hat{\Delta}_0^{(\mathrm{int})} = -2\left(\delta\beta_1^{(\mathrm{int})} + \gamma\beta_0^{(\mathrm{int})}\right) + \partial_\mu k_0^\mu. \tag{288}$$

By a trivial redefinition, (286) can always be replaced with

$$\gamma\beta_2^{(\mathrm{int})} = 0. \tag{289}$$

Analyzing the expression of $\hat{\Delta}_1^{(\mathrm{int})}$ in (283) we observe that it can be written like in (287) if the quantity

$$\hat{\chi} = \left(k_A f^A_{BC} - k_B k_C\right)\left(\psi^{*\mu}\left(\partial^\nu\psi_\mu\right)\eta^{B\rho}\partial_{[\nu}\eta^C_{\rho]} + \frac{1}{4}\left(\psi^{*[\mu}\psi^{\nu]} + \frac{1}{2}\psi^{*\sigma}\gamma^{\mu\nu}\psi_\sigma\right)\partial_{[\mu}\eta^B_{\rho]}\partial_{[\nu}\eta^C_{\lambda]}\sigma^{\rho\lambda}\right) \tag{290}$$

can be put in the form

$$\hat{\chi} = \delta\hat{\varphi} + \gamma\hat{\omega} + \partial_\mu j^\mu. \tag{291}$$

Assume that (291) holds. Then, by applying δ on this equation we infer

$$\delta\hat{\chi} = \gamma\left(-\delta\hat{\omega}\right) + \partial_\mu\left(\delta j^\mu\right). \tag{292}$$

On the other hand, if we use the concrete expression (290) of $\hat{\chi}$, by direct computation we are led to

$$\begin{aligned}\delta\hat{\chi} = {} & \gamma\left(\frac{1}{2}\left(k_A f^A_{BC} - k_B k_C\right) \times \delta\left(\psi^{*\rho}\psi_\rho\eta^B_\nu\left(\partial_\mu h^{C\mu\nu} - \partial^\nu h^C\right)\right)\right) \\ & + \partial^\mu\left(\frac{1}{2}\left(k_A f^A_{BC} - k_B k_C\right)\delta\left(\psi^{*\rho}\psi_\rho\eta^{B\nu}\partial_{[\mu}\eta^C_{\nu]}\right)\right) \\ & + \gamma\left(\frac{\mathrm{i}}{4}\left(k_A f^A_{BC} - k_B k_C\right)\left(\left(\bar{\psi}_\beta\gamma^{\alpha\beta\sigma}\left(\partial^\mu\psi_\sigma\right)h^{B\rho}_\alpha\right.\right.\right. \\ & - \left(\bar{\psi}_\beta\gamma^{\alpha\beta[\mu}\psi^{\nu]} - \bar{\psi}^\mu\gamma^\alpha\psi^\nu - \sigma^{\alpha[\mu}\bar{\psi}^{\nu]}\gamma^\sigma\psi_\sigma\right) \\ & \left.\times\,\sigma^{\rho\lambda}\partial_{[\nu}h^B_{\lambda]\alpha}\right)\partial_{[\mu}\eta^C_{\rho]} \\ & \left.\left. -2\bar{\psi}_\beta\gamma^{\alpha\beta\mu}\left(\partial^\nu\psi_\mu\right)\eta^{B\rho}\partial_{[\nu}h^C_{\rho]\alpha}\right)\right) \\ & + \partial_\alpha\left(\frac{\mathrm{i}}{2}\left(k_A f^A_{BC} - k_B k_C\right)\left(\bar{\psi}_\beta\gamma^{\alpha\beta\sigma}\left(\partial^\mu\psi_\sigma\right)\eta^{B\rho}\right.\right. \\ & - \frac{1}{4}\left(\bar{\psi}_\beta\gamma^{\alpha\beta[\mu}\psi^{\nu]} - \bar{\psi}^\mu\gamma^\alpha\psi^\nu - \sigma^{\alpha[\mu}\bar{\psi}^{\nu]}\gamma^\sigma\psi_\sigma\right) \\ & \left.\left.\times\,\sigma^{\rho\lambda}\partial_{[\nu}\eta^B_{\lambda]}\right)\partial_{[\mu}\eta^C_{\rho]}\right).\end{aligned} \tag{293}$$

The right-hand side of (293) can be written like in the right-hand side of (292) if the following conditions are simultaneously fulfilled

$$\frac{\mathrm{i}}{4}\left(k_A f^A_{BC} - k_B k_C\right)\left\{\left[\bar{\psi}_\beta\gamma^{\alpha\beta\sigma}\left(\partial^\mu\psi_\sigma\right)h^\rho_\alpha - \left(\bar{\psi}_\beta\gamma^{\alpha\beta[\mu}\psi^{\nu]} - \bar{\psi}^\mu\gamma^\alpha\psi^\nu - \sigma^{\alpha[\mu}\bar{\psi}^{\nu]}\gamma^\sigma\psi_\sigma\right)\sigma^{\rho\lambda}\partial_{[\nu}h^B_{\lambda]\alpha}\right]\partial_{[\mu}\eta^C_{\rho]} - 2\bar{\psi}_\beta\gamma^{\alpha\beta\mu}\left(\partial^\nu\psi_\mu\right)\eta^{B\rho}\partial_{[\nu}h^C_{\rho]\alpha}\right\} = -\delta\hat{\omega}', \tag{294}$$

$$\frac{\mathrm{i}}{2}\left(k_A f^A_{BC} - k_B k_C\right)\left(\bar{\psi}_\beta\gamma^{\alpha\beta\sigma}\left(\partial^\mu\psi_\sigma\right)\eta^{B\rho} - \frac{1}{4}\left(\bar{\psi}_\beta\gamma^{\alpha\beta[\mu}\psi^{\nu]} - \bar{\psi}^\mu\gamma^\alpha\psi^\nu - \sigma^{\alpha[\mu}\bar{\psi}^{\nu]}\gamma^\sigma\psi_\sigma\right)\sigma^{\rho\lambda}\partial_{[\nu}\eta^B_{\lambda]}\right)\partial_{[\mu}\eta^C_{\rho]} = \delta j'^\mu. \tag{295}$$

However, from the action of δ on the BRST generators we observe that none of $h^{A\mu\beta}$, $\partial_{[\alpha}h^A_{\beta]\mu}$, η^A_β, or $\partial_{[\lambda}\eta^A_{\beta]}$ are δ-exact. In consequence, the relations (294)–(295) hold if the equations

$$\bar{\psi}_\beta\gamma^{\alpha\beta\sigma}\left(\partial_\mu\psi_\sigma\right) = \delta\Omega^\alpha_{\ \mu} \tag{296}$$

and

$$\bar{\psi}_\beta\gamma^{\alpha\beta[\mu}\psi^{\nu]} - \bar{\psi}^\mu\gamma^\alpha\psi^\nu - \sigma^{\alpha[\mu}\bar{\psi}^{\nu]}\gamma^\sigma\psi_\sigma = \delta\Gamma^{\mu\nu\alpha} \tag{297}$$

take place simultaneously. The last equations are precisely (157) and (158), respectively. Due to the fact that they do not involve (Pauli–Fierz) collection indices, some arguments identical to those employed in Sect. 4.3 ensure that (296) and (297) cannot be satisfied. As a consequence, $\hat{\chi}$ must vanish, which further implies that

$$k_D f^D_{AB} - k_A k_B = 0. \tag{298}$$

Using (298) and (279) we obtain that for $A \neq B$

$$k_A k_B = 0, \tag{299}$$

which shows that the Rarita–Schwinger field can couple to only one graviton, so the assertion from the beginning of this section is finally proved.

7 Conclusion

To conclude, in this paper we have investigated the couplings between a collection of massless spin-two fields (described in the free limit by a sum of Pauli–Fierz actions) and a massive Rarita–Schwinger field using a powerful setting based on local BRST cohomology. Initially, we showed that if we decompose the metric like $g_{\mu\nu} = \sigma_{\mu\nu} + gh_{\mu\nu}$, then we can couple the massive Rarita–Schwinger field to $h_{\mu\nu}$ in the space of formal series with the maximum derivative order equal to one in $h_{\mu\nu}$. The interacting Lagrangian $\mathcal{L}^{(\mathrm{int})}$ obtained here contains, besides the standard minimal couplings, also three types of non-minimal couplings, which are not discussed in the literature, but are nevertheless consistent with the gauge symmetries of the Lagrangian $\mathcal{L}_2 + \mathcal{L}^{(\mathrm{int})}$, where $\mathcal{L}_2$ is the full spin-two

Lagrangian in the vierbein formulation. Next, we have proved, under the hypotheses of locality, smoothness of the interactions in the coupling constant, Poincaré invariance, (background) Lorentz invariance and the preservation of the number of derivatives on each field, that there are no consistent cross-interactions among different gravitons in the presence of a massive Rarita–Schwinger field if the metric in internal space is positively defined.

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Appendix A: Main conventions and properties of the γ-matrices

Here, we collect the main conventions and properties of the representation of the γ-matrices employed in this paper. We work with the charge conjugation matrix

$$\mathcal{C} = -\gamma_0 \tag{A.1}$$

and with that representation of the Clifford algebra

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\sigma_{\mu\nu}\mathbf{1}\,, \tag{A.2}$$

for which all the γ-matrices are purely imaginary. In addition, γ_0 is Hermitian and antisymmetric, while $(\gamma_i)_{i=\overline{1,3}}$ are anti-Hermitian and symmetric. We take a basis in the space of spinor matrices of the form

$$\mathbf{1}\,,\quad \gamma_\mu\,,\quad \gamma_{\mu_1\mu_2}\,,\quad \gamma_{\mu_1\mu_2\mu_3}\,,\quad \gamma_{\mu_1\mu_2\mu_3\mu_4}\,, \tag{A.3}$$

where

$$\gamma_{\mu_1\cdots\mu_k} = \frac{1}{k!}\sum_{\sigma\in S_k}(-)^\sigma\,\gamma_{\mu_{\sigma(1)}}\gamma_{\mu_{\sigma(2)}}\cdots\gamma_{\mu_{\sigma(k)}}\,. \tag{A.4}$$

In the above definition, S_k is the set of permutations of $\{1,2,\ldots,k\}$ and $(-)^\sigma$ denotes the signature of a given permutation σ. This means that any 4×4 matrix M with purely spinor indices can be expressed in terms of the matrices (A.3) via

$$M = \frac{1}{4}\sum_{k=0}^{4}(-)^{k(k-1)/2}\,\frac{1}{k!}\mathrm{Tr}\left(\gamma^{\mu_1\cdots\mu_k}M\right)\gamma_{\mu_1\cdots\mu_k}\,. \tag{A.5}$$

We list below some Fierz identities that are useful for the construction of consistent interactions between the Pauli–Fierz field and the massive Rarita–Schwinger spinor. They provide the products of the various elements from (A.3) in terms of their linear combinations

$$\gamma_{\mu\nu}\gamma^\rho = -\delta^\rho_{[\mu}\gamma_{\nu]} + \gamma_{\mu\nu}{}^\rho\,, \tag{A.6}$$

$$\gamma_{\mu\nu}\gamma^{\rho\lambda} = -\delta^\rho_{[\mu}\delta^\lambda_{\nu]}\mathbf{1} - \delta^{[\rho}_{[\mu}\gamma_{\nu]}{}^{\lambda]} + \gamma_{\mu\nu}{}^{\rho\lambda}\,, \tag{A.7}$$

$$\gamma_{\mu\nu}\gamma^{\rho\lambda\sigma} = -\delta^{[\rho}_{\mu}\delta^\lambda_\nu\gamma^{\sigma]} - \delta^{[\rho}_{[\mu}\gamma_{\nu]}{}^{\lambda\sigma]}\,, \tag{A.8}$$

$$\gamma_{\mu\nu}\gamma^{\rho\lambda\sigma\xi} = -\delta^{[\rho}_{\mu}\delta^\lambda_\nu\gamma^{\sigma\xi]}\,, \tag{A.9}$$

$$\gamma_{\mu\nu\rho}\gamma^\alpha = \delta^\alpha_{[\mu}\gamma_{\nu\rho]} + \gamma_{\mu\nu\rho}{}^\alpha\,, \tag{A.10}$$

$$\gamma_{\mu\nu\rho}\gamma^{\alpha\beta\gamma} = -\delta^{[\alpha}_{\mu}\delta^\beta_\nu\delta^{\gamma]}_\rho\mathbf{1} - \delta^{[\alpha}_{[\mu}\delta^\beta_\nu\gamma_{\rho]}{}^{\gamma]}\,. \tag{A.11}$$

Moreover, in the chosen representation of the γ-matrices the elements of the basis (A.3) display the following symmetry/antisymmetry properties:

$$\gamma_0\gamma_\mu\,,\quad \gamma_0\gamma_{\mu\nu} \tag{A.12}$$

are symmetric and

$$\gamma_0\gamma_{\mu\nu\rho}\,,\quad \gamma_0\gamma_{\mu\nu\rho\lambda}\,,\quad \gamma_0\gamma_5 \tag{A.13}$$

are antisymmetric. If we take $\gamma_5 = \mathrm{i}\gamma_0\gamma_1\gamma_2\gamma_3$ and work with $\varepsilon^{0123} = -\varepsilon_{0123} = 1$, then

$$\gamma^{\mu\nu\rho\lambda} = \varepsilon^{\mu\nu\rho\lambda}\gamma^0\gamma^1\gamma^2\gamma^3 = \mathrm{i}\varepsilon^{\mu\nu\rho\lambda}\gamma_5\,, \tag{A.14}$$

$$\gamma_{\mu\nu\rho\lambda} = -\varepsilon_{\mu\nu\rho\lambda}\gamma_0\gamma_1\gamma_2\gamma_3 = \mathrm{i}\varepsilon_{\mu\nu\rho\lambda}\gamma_5\,. \tag{A.15}$$

Appendix B: Proof of some assertions made in Sect. 4.2

Initially, we show that our statement from footnote [3] is indeed valid. The terms linear in the Pauli–Fierz antifield $h^{*\mu\nu}$ that can in principle be added to $a_1^{(\mathrm{int})}$ have the generic form

$$\tilde{a}_1^{(\mathrm{int})} = h^{*\mu\nu}\left(M^\rho_{\mu\nu}\eta_\rho + M^{\rho\lambda}_{\mu\nu}\partial_{[\rho}\eta_{\lambda]}\right) \equiv \tilde{a}_1'^{(\mathrm{int})} + \tilde{a}_1''^{(\mathrm{int})}\,, \tag{B.1}$$

where $M^\rho_{\mu\nu}$ and $M^{\rho\lambda}_{\mu\nu}$ are bosonic, real, gauge-invariant functions. Imposing that (B.1) satisfies the requirements i)–ii) from Sect. 4.2, the functions $M^\rho_{\mu\nu}$ and $M^{\rho\lambda}_{\mu\nu}$ are restricted to depend at most on the undifferentiated Rarita–Schwinger field. The consistency equation for $\tilde{a}_1^{(\mathrm{int})}$ in antighost number zero

$$\delta\tilde{a}_1^{(\mathrm{int})} + \gamma\tilde{a}_0^{(\mathrm{int})} = \partial_\mu\tilde{j}_0^{(\mathrm{int})} \tag{B.2}$$

is independent of that for $a_1^{(\mathrm{int})}$ of the form (57) since the former piece produces in $\tilde{a}_0^{(\mathrm{int})}$ components quadratic in the Pauli–Fierz field, while the latter introduces in $a_0^{(\mathrm{int})}$ terms linear in $h_{\mu\nu}$. Moreover, the consistency equation of $\tilde{a}_1'^{(\mathrm{int})}$ is independent of that implying $\tilde{a}_1''^{(\mathrm{int})}$ due to the different number of derivatives contained in these two types of terms, so (B.2) is equivalent to the equations

$$\delta\tilde{a}_1'^{(\mathrm{int})} + \gamma\tilde{a}_0'^{(\mathrm{int})} = \partial_\mu\tilde{j}_0'^{(\mathrm{int})}\,, \tag{B.3}$$

$$\delta\tilde{a}_1''^{(\mathrm{int})} + \gamma\tilde{a}_0''^{(\mathrm{int})} = \partial_\mu\tilde{j}_0''^{(\mathrm{int})}\,. \tag{B.4}$$

Now, we prove that (B.1) is not consistent in antighost number zero, i.e. there are no solutions $\tilde{a}_0'^{(\mathrm{int})}$ or $\tilde{a}_0''^{(\mathrm{int})}$ to (B.3)–(B.4). To this end we use the fact that the linearized Einstein tensor (17) can be written as

$$H^{\mu\nu} = \partial_\alpha \partial_\beta \phi^{\mu\alpha\nu\beta} \,, \tag{B.5}$$

with

$$\phi^{\mu\alpha\nu\beta} = \frac{1}{2}\left(-h^{\mu\nu}\sigma^{\alpha\beta} + h^{\alpha\nu}\sigma^{\mu\beta} + h^{\mu\beta}\sigma^{\alpha\nu} - h^{\alpha\beta}\sigma^{\mu\nu} \right. \\ \left. + h\left(\sigma^{\mu\nu}\sigma^{\alpha\beta} - \sigma^{\mu\beta}\sigma^{\alpha\nu}\right)\right) \,. \tag{B.6}$$

By direct computation, we find that

$$\begin{aligned} \delta\tilde{a}_1'^{(\mathrm{int})} &= -2\partial_\alpha\partial_\beta\phi^{\mu\alpha\nu\beta} M^\rho_{\mu\nu}\eta_\rho \\ &= \partial_\alpha\left(-2\left(\partial_\beta\phi^{\mu\alpha\nu\beta}\right) M^\rho_{\mu\nu}\eta_\rho\right) \\ &\quad + \partial_\beta\left(2\phi^{\mu\alpha\nu\beta}\partial_\alpha\left(M^\rho_{\mu\nu}\eta_\rho\right)\right) \\ &\quad + \phi^{\mu\alpha\nu\beta}\partial_{[\mu}M^\rho_{\alpha]\nu}\partial_{[\beta}\eta_{\rho]} + \frac{1}{2}\phi^{\mu\alpha\nu\beta}\partial_{[\mu}M^\rho_{\alpha][\nu,\beta]}\eta_\rho \\ &\quad + \gamma\left(\phi^{\mu\alpha\nu\beta}\left(\partial_{[\mu}M^\rho_{\alpha]\nu}h_{\beta\rho} - 2M^\rho_{\mu\nu}\overset{(1)}{\Gamma}_{\rho\alpha\beta}\right)\right) \\ &\quad - \left(\gamma\phi^{\mu\alpha\nu\beta}\right)\left(\partial_{[\mu}M^\rho_{\alpha]\nu}h_{\beta\rho} - 2M^\rho_{\mu\nu}\overset{(1)}{\Gamma}_{\rho\alpha\beta}\right) \,, \end{aligned} \tag{B.7}$$

where

$$\overset{(1)}{\Gamma}_{\rho\alpha\beta} = \frac{1}{2}\left(\partial_\alpha h_{\beta\rho} + \partial_\beta h_{\alpha\rho} - \partial_\rho h_{\alpha\beta}\right) \,. \tag{B.8}$$

Comparing (B.7) with (B.3) and observing that the term in (B.7) involving $\left(\gamma\phi^{\mu\alpha\nu\beta}\right)$ comprises the symmetric derivatives $\partial_{(\beta}\eta_{\rho)}$, it follows that this piece, which is constrained to contribute to a full divergence, can only realize this task together with the part proportional with $\partial_{[\mu}M^\rho_{\alpha][\nu,\beta]}$. Accordingly, the γ-exactness modulo d of the right-hand side of (B.7), which is demanded by (B.3), requires that the functions $M^\rho_{\mu\nu}$ are subject to the equations

$$\partial_{[\mu}M^\rho_{\alpha]\nu} = 0 \,, \tag{B.9}$$

possessing the trivial solution

$$M^\rho_{\alpha\nu} = 0 \,, \tag{B.10}$$

since $M^\rho_{\alpha\nu}$ are derivative-free (they depend only on the undifferentiated spinor-vector ψ_μ). In an identical manner, starting with

$$\begin{aligned} \delta\tilde{a}_1''^{(\mathrm{int})} &= -2\partial_\alpha\partial_\beta\phi^{\mu\alpha\nu\beta} M^{\rho\lambda}_{\mu\nu}\partial_{[\rho}\eta_{\lambda]} \\ &= \partial_\alpha\left(-2\left(\partial_\beta\phi^{\mu\alpha\nu\beta}\right) M^{\rho\lambda}_{\mu\nu}\partial_{[\rho}\eta_{\lambda]}\right) \\ &\quad + \partial_\beta\left(2\phi^{\mu\alpha\nu\beta}\partial_\alpha\left(M^{\rho\lambda}_{\mu\nu}\partial_{[\rho}\eta_{\lambda]}\right)\right) \\ &\quad + \frac{1}{2}\phi^{\mu\alpha\nu\beta}\partial_{[\mu}M^{\rho\lambda}_{\alpha][\nu,\beta]}\partial_{[\rho}\eta_{\lambda]} \\ &\quad + \gamma\left(2\phi^{\mu\alpha\nu\beta}\left(\partial_{[\mu}M^{\rho\lambda}_{\alpha]\nu}\partial_{[\rho}h_{\lambda]\beta} - M^\rho_{\mu\nu}\partial_\alpha\partial_{[\rho}h_{\lambda]\beta}\right)\right) \\ &\quad - 2\left(\gamma\phi^{\mu\alpha\nu\beta}\right) \\ &\quad \times\left(\partial_{[\mu}M^{\rho\lambda}_{\alpha]\nu}\partial_{[\rho}h_{\lambda]\beta} - M^\rho_{\mu\nu}\partial_\alpha\partial_{[\rho}h_{\lambda]\beta}\right) \,, \end{aligned} \tag{B.11}$$

we argue that the functions $M^{\rho\lambda}_{\mu\nu}$ must obey the equations

$$\partial_{[\mu}M^{\rho\lambda}_{\alpha][\nu,\beta]} = 0 \,, \tag{B.12}$$

which, due to the fact that $M^{\rho\lambda}_{\mu\nu}$ are derivative-free, possess only the trivial solution

$$M^{\rho\lambda}_{\mu\nu} = 0 \,. \tag{B.13}$$

If we substitute the results (B.10) and (B.13) into (B.1), we conclude that there is no term linear in the Pauli–Fierz antifield $h^{*\mu\nu}$ that can be added to $a_1^{(\mathrm{int})}$ such as to give a consistent component of antighost number zero in the first-order deformation of the solution to the master equation.

Finally, we show that we can always make the functions c_1, c_2, and c_3 from (57) vanish via adding some trivial terms and making some redefinitions of the functions $\bar{N}^{\rho\lambda\sigma}{}_\mu$. In view of this, we insert (65) in (57), such that the part from $a_1^{(\mathrm{int})}$ proportional with c_1, c_2, or c_3 reads as

$$\begin{aligned} T\left(c_1\,, c_2\,, c_3\right) = \Big[& c_1\left(\psi^{*\lambda}\gamma^\mu\psi_\mu - \frac{1}{2}\psi^*_\mu\gamma^{\mu\nu\lambda}\psi_\nu\right) \\ & + c_2\left(\psi^{*\mu}\gamma^\lambda\psi_\mu - \psi^*_\mu\gamma^{\mu\nu\lambda}\psi_\nu\right) \\ & + c_3\left(\psi^{*\mu}\gamma_\mu\psi^\lambda - \frac{3}{2}\psi^*_\mu\gamma^{\mu\nu\lambda}\psi_\nu\right)\Big]\eta_\lambda \,. \end{aligned} \tag{B.14}$$

Based on the second definition in (12) related to the Koszul–Tate differential and on the Fierz identities from the previous appendix section, we obtain that

$$\begin{aligned} \delta\left(\psi^{*\lambda}\gamma_\mu\bar{\psi}^{*\mu}\right) &= -4m\psi^{*\lambda}\gamma^\mu\psi_\mu + m\psi^{*\mu}\gamma^\lambda\psi_\mu \\ &\quad + m\psi^*_\mu\gamma^{\mu\nu\lambda}\psi_\nu + \mathrm{i}\left(3\psi^{*\lambda}\gamma^{\mu\nu} + \psi^{*[\mu}\gamma^{\nu]\lambda}\right) \\ &\quad \times\partial_\mu\psi_\nu + \mathrm{i}\psi^*_\mu\gamma^{\mu\nu\rho\lambda}\partial_\nu\psi_\rho \,, \end{aligned} \tag{B.15}$$

$$\begin{aligned} \delta\left(\psi^*_\mu\gamma^\lambda\bar{\psi}^{*\mu}\right) &= -2m\psi^{*\lambda}\gamma^\mu\psi_\mu + 2m\psi^*_\mu\gamma^\mu\psi^\lambda \\ &\quad - 2m\psi^*_\mu\gamma^{\mu\nu\lambda}\psi_\nu \\ &\quad + 2\mathrm{i}\left(\psi^{*\lambda}\gamma^{\mu\nu}\partial_\mu\psi_\nu + \psi^{*\mu}\gamma_{\rho\mu}\partial^{[\lambda}\psi^{\rho]}\right) \\ &\quad + 2\mathrm{i}\psi^*_\mu\gamma^{\mu\nu\rho\lambda}\partial_\nu\psi_\rho \,, \end{aligned} \tag{B.16}$$

$$\begin{aligned} \delta\left(\psi^*_\mu\gamma^{\mu\nu\lambda}\bar{\psi}^*_\nu\right) &= 4m\psi^*_\mu\gamma^\mu\psi^\lambda - 4m\psi^{*\mu}\gamma^\lambda\psi_\mu - 2m\psi^*_\mu\gamma^{\mu\nu\lambda}\psi_\nu \\ &\quad + 4\mathrm{i}\psi^*_\mu\partial^{[\mu}\psi^{\lambda]} + 2\mathrm{i}\psi^{*\mu}\gamma^{\lambda\nu}\partial_{[\mu}\psi_{\nu]} \\ &\quad - 2\mathrm{i}\psi^{*\mu}\gamma_{\mu\nu}\partial^{[\lambda}\psi^{\nu]} \,. \end{aligned} \tag{B.17}$$

Relying on the above results, we can rewrite the three terms present in (B.14) in the form

$$c_1\left(\psi^{*\lambda}\gamma^\mu\psi_\mu - \frac{1}{2}\psi^*_\mu\gamma^{\mu\nu\lambda}\psi_\nu\right)\eta_\lambda \\ = s\left[\frac{c_1}{12m}\left(4\psi^{*\rho}\gamma^\mu\bar{\psi}^*_\mu - 2\psi^*_\mu\gamma^\rho\bar{\psi}^{*\mu} + \psi^*_\mu\gamma^{\mu\nu\rho}\bar{\psi}^*_\nu\right)\eta_\rho\right]$$

$$+\frac{\mathrm{i}c_1}{3m}\left[\left(2\psi^{*\lambda}\gamma^{\mu\nu}+\frac{1}{2}\psi^{*[\mu}\gamma^{\nu]\lambda}\right)\partial_\mu\psi_\nu\right.$$
$$\left.+\frac{1}{2}\psi^{*\mu}\gamma_{\mu\rho}\partial^{[\lambda}\psi^{\rho]}+\psi^*_\mu\partial^{[\mu}\psi^{\lambda]}+\frac{1}{2}\psi^*_\mu\gamma^{\mu\nu\rho\lambda}\partial_\nu\psi_\rho\right]\eta_\lambda\,, \tag{B.18}$$

$$c_2\left(\psi^{*\mu}\gamma^\lambda\psi_\mu-\psi^*_\mu\gamma^{\mu\nu\lambda}\psi_\nu\right)\eta_\lambda$$
$$=s\left[\frac{c_2}{3m}\left(\psi^{*\rho}\gamma^\mu\bar{\psi}^*_\mu-2\psi^*_\mu\gamma^\rho\bar{\psi}^{*\mu}+\psi^*_\mu\gamma^{\mu\nu\rho}\bar{\psi}^*_\nu\right)\eta_\rho\right]$$
$$+\frac{\mathrm{i}c_2}{3m}\left[-\left(\psi^{*\lambda}\gamma^{\mu\nu}+\psi^{*[\mu}\gamma^{\nu]\lambda}\right)\partial_\mu\psi_\nu\right.$$
$$\left.+2\psi^{*\mu}\gamma_{\mu\rho}\partial^{[\lambda}\psi^{\rho]}+4\psi^*_\mu\partial^{[\mu}\psi^{\lambda]}-3\psi^*_\mu\gamma^{\mu\nu\rho\lambda}\partial_\nu\psi_\rho\right]\eta_\lambda\,, \tag{B.19}$$

$$c_3\left(\psi^{*\mu}\gamma_\mu\psi^\lambda-\frac{3}{2}\psi^*_\mu\gamma^{\mu\nu\lambda}\psi_\nu\right)\eta_\lambda$$
$$=s\left[\frac{c_3}{12m}\left(4\psi^{*\rho}\gamma^\mu\bar{\psi}^*_\mu-8\psi^*_\mu\gamma^\rho\bar{\psi}^{*\mu}+\psi^*_\mu\gamma^{\mu\nu\rho}\bar{\psi}^*_\nu\right)\eta_\rho\right]$$
$$+\frac{\mathrm{i}c_3}{12m}\left[\left(-4\psi^{*\lambda}\gamma^{\mu\nu}+2\psi^{*[\mu}\gamma^{\nu]\lambda}\right)\partial_\mu\psi_\nu\right.$$
$$\left.+14\psi^{*\mu}\gamma_{\mu\rho}\partial^{[\lambda}\psi^{\rho]}+4\psi^*_\mu\partial^{[\mu}\psi^{\lambda]}-12\psi^*_\mu\gamma^{\mu\nu\rho\lambda}\partial_\nu\psi_\rho\right]\eta_\lambda\,. \tag{B.20}$$

By adding the relations (B.18)–(B.20), we observe that $T(c_1, c_2, c_3)$ can be made to vanish by adding some s-exact terms to the first-order deformation $a^{(\mathrm{int})}$ and by appropriately redefining the functions $\bar{N}^{\rho\lambda\sigma}{}_{\mu}$.

References

1. C. Cutler, R.M. Wald, Class. Quantum Grav. **4**, 1267 (1987)
2. R.M. Wald, Class. Quant. Grav. **4**, 1279 (1987)
3. A. Hindawi, B.A. Ovrut, D. Waldram, Phys. Rev. D **53**, 5583 (1996) [hep-th/9509142]
4. S.C. Anco, Annals Phys. **270**, 52 (1998)
5. S.N. Gupta, Phys. Rev. **96**, 1683 (1954)
6. R.H. Kraichnan, Phys. Rev. **98**, 1118 (1955)
7. S. Weinberg, Phys. Rev. **138**, 988 (1965)
8. S. Deser, Gen. Rel. Grav. **1**, 9 (1970) [gr-qc/0411023]
9. D.G. Boulware, S. Deser, Annals Phys. **89**, 193 (1975)
10. J. Fang, C. Fronsdal, J. Math. Phys. **20**, 2264 (1979)
11. F.A. Berends, G.J.H. Burgers, H. Van Dam, Z. Phys. C **24**, 247 (1984)
12. R.M. Wald, Phys. Rev. D **33**, 3613 (1986)
13. R.P. Feynman, F.B. Morinigo, W.G. Wagner, Feynman Lectures on Gravitation (ed. by B. Hatfield, Addison-Wesley, Reading, 1995)
14. N. Boulanger, M. Esole, Class. Quantum Grav. **19**, 2107 (2002) [gr-qc/0110072]
15. C. Bizdadea, E.M. Cioroianu, A.C. Lungu, S.O. Saliu, JHEP **0502**, 016 (2005)
16. N. Boulanger, T. Damour, L. Gualtieri, M. Henneaux, Nucl. Phys. B **597**, 127 (2001) [hep-th/0007220]
17. I.A. Batalin, G.A. Vilkovisky, Phys. Lett. B **102**, 27 (1981)
18. I.A. Batalin, G.A. Vilkovisky, Phys. Rev. D **28**, 2567 (1983); Erratum-ibid. D **30**, 508 (1984)
19. I.A. Batalin, G.A. Vilkovisky, J. Math. Phys. **26**, 172 (1985)
20. M. Henneaux, Nucl. Phys. **B** (Proc. Suppl.) **18A**, 47 (1990)
21. M. Henneaux, C. Teitelboim, Quantization of Gauge Systems (Princeton University Press, Princeton, 1992)
22. G. Barnich, M. Henneaux, Phys. Lett. B **311**, 123 (1993) [hep-th/9304057]
23. G. Barnich, F. Brandt, M. Henneaux, Commun. Math. Phys. **174**, 57 (1995) [hep-th/9405109]; Phys. Rept. **338**, 439 (2000) [hep-th/0002245]
24. W. Pauli, M. Fierz, Helv. Phys. Acta **12**, 297 (1939); M. Fierz, W. Pauli, Proc. Roy. Soc. Lond. A **173**, 211 (1939)
25. W. Rarita, J. Schwinger, Phys. Rev. **60**, 61 (1941)
26. M. Henneaux, Commun. Math. Phys. **140**, 1 (1991)
27. W. Siegel, hep-th/9912205